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LECTURE NOTES ON LIGHT

CAMBRIDGE UNIVERSITY PRESS
C. F. CLAY, MANAGER

LONDON
FETTER LANE, E.C. 4

EDINBURGH
100 PRINCES STREET

LONDON: H. K. LEWIS AND CO., LTD., 136 Gower Street, W.C. 1
LONDON: WILLIAM WESLEY AND SON, 28 Essex Street, Strand, W.C. 2
NEW YORK: G. P. PUTNAM'S SONS
BOMBAY, CALCUTTA, MADRAS: MACMILLAN AND CO., LTD.
TORONTO: J. M. DENT AND SONS, LTD.
TOKYO: THE MARUZEN-KABUSHIKI-KAISHA

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LECTURE NOTES ON LIGHT

BY

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Assistant Master at Gresham's School, Holt

WITH DIAGRAMS

152780
9/10/19

Cambridge:
at the University Press
1917

PREFACE

THIS book marks in some ways a new departure and differs in certain important respects from the usual type of School Science Manual. It is not a text-book in the ordinary sense of the term, but, as the title indicates, a collection of lecture notes.

The book is based on the belief that the study of Physics should be conducted by means of experimental lectures, combined with practical work in the laboratory, and that the lectures should be summarised in the form of notes. Anyone who sets out to learn any branch of Physics, merely with the aid of a text-book, is likely to discover that he has undertaken an impossible task. To get a real and thorough grasp of his subject some form of notes is absolutely indispensable, and it is of the first importance that these notes should be correctly drawn up and well arranged. It is with the idea of supplying this need that these notes have been compiled, and no pains have been spared to make them as complete as possible.

If the importance of having good notes is once accepted, the question then arises as to the form that they should take. Should they be written or printed notes? For a good many years notes on Physics were dictated to the boys at Gresham's School. It was found, however, that this took up a considerable amount of valuable time, which could be saved by having the notes printed and copies supplied to the boys. The result has been a gain of fifty per cent. in the amount of ground covered. At the same time it has been found that the subject matter has been acquired by the boys just as well as through the dictated notes, perhaps even better.

In the study of Light the drawing of diagrams forms a by no means inconsiderable portion of the work and training. A special feature of the book will be found to be the space provided for this purpose. It will be noticed that the left-hand pages have been kept entirely blank so as to enable boys to insert their own diagrams.

The present volume is a reprint, with certain corrections and modifications, of the original one. An edition, however, is also being published complete with diagrams. This is designed to serve mainly as a key, for the use of Masters, to the diagrams referred to in the text; it is hoped that it may also be found useful for boys who are compelled to work rapidly through the subject, and have not sufficient time to draw the diagrams for themselves.

The book is intended to supplement, but not to replace, lectures, and it is meant to be used in the following manner. The Master lectures on the lines of the notes and draws the diagrams on the board. The boys enter the diagrams in their books, either during the hour or in preparation, whichever is found to be the more convenient.

In order to draw attention to certain laws, statements etc., which it is important to learn precisely, the author has adopted the plan of putting the letters "Def.," implying "Definition," before those portions of the notes which, in his opinion, ought to be learnt "by heart."

It would be impossible to let the book go to print without a word of grateful thanks to the many Gresham boys whose keenness and enthusiasm have made the compilation of these notes such a pleasure to the author. A special word of thanks is also due to those who have helped, whether as proof-readers or in other ways, to render the work less imperfect. It is, perhaps, somewhat invidious to mention names where so many have done valuable work, but it seems only right to say that the reproduction of the diagrams owes very much to M. J. Gregory and P. D. Ballance, without whose help it would have been very difficult to provide the illustrations required.

J. R. E.

August 1917.

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INTRODUCTION

Nature of Light

In a dark room we say it is dark, because the retina of the eye is unaffected by any stimulus. If a match is struck, some stimulus affects the retina of the eye and we say there is light. The lighted match is a source of energy, and this energy is transmitted to the retina of the eye and gives us the sensation of vision.

In 1676 Römer found that light travelled with a velocity of 186,000 miles per sec. How is this energy or light transmitted?

Two Theories of the Transmission of Light

I. Corpuscular or Emission Theory

A luminous body continually emits small particles or corpuscles in all directions. These travel through space with the velocity of light and give up their energy to the retina of the eye on coming in contact with it.

The objection to this theory is that, if the corpuscles travelled with this enormous velocity, they would possess a considerable momentum, of which there is no evidence.

II. Wave Theory

A luminous body is in a state of vibration and causes waves in the "aether," which is a medium pervading all space. These waves travel out from the body in all directions with the velocity of light, and, falling upon the retina of the eye, produce the sensation of vision.

This theory is the one that is now in vogue.

Definition of some Terms used in Light

Def.

"A luminous body is one that emits light."

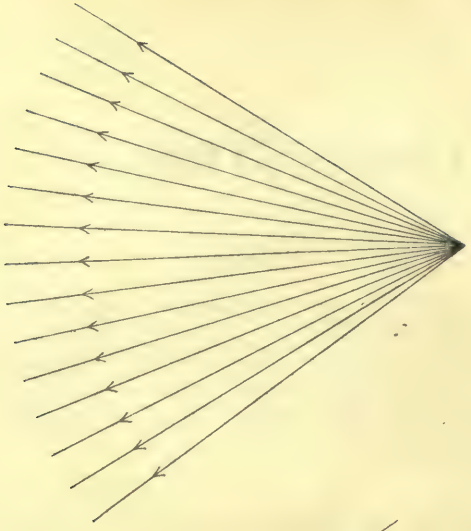
e.g. the sun, a candle, an arc lamp, a fixed star, etc.

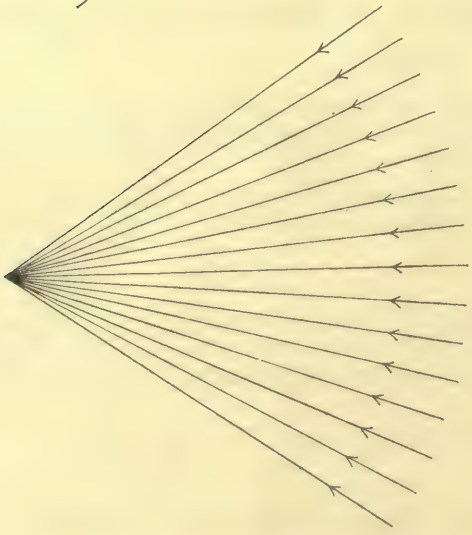
"A non-luminous or illuminated body is one that does *not* emit light, but is visible by means of light which it receives from other bodies and reflects to the eye."

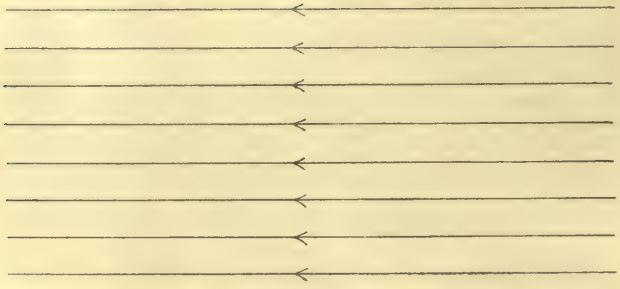
e.g. most of the objects in a room or out of doors, the moon, the planets, etc.

"A medium is a substance through which light can be transmitted."

e.g. air, glass, water, etc.







"A transparent substance is one through which light can be transmitted and through which objects can be distinctly seen."

e.g. air, glass, water, etc.

"A translucent substance is one through which light can be transmitted, but through which objects cannot be distinctly seen."

e.g. ground glass, holland blinds, oiled paper, etc.

"An opaque substance is one through which light cannot be transmitted."

Rectilinear Propagation of Light

Evidence that Light travels in straight lines

1. A beam of sunlight entering a room, seen by means of the particles of dust, has sharp and straight edges. These edges are parallel owing to the great distance that the sun is away.

2. The edges of a beam of light from an optical lantern or search-light are sharp and straight.

3. Inability to see round corners, or the immediate disappearance of a point of light on interposing an opaque body.

4. Pinhole Camera.

5. Formation of shadows.

Ray of Light

Def.

"Since light travels in straight lines, any one of these straight lines is called a ray of light."

A ray has no material existence, but indicates the direction in which the light is proceeding.

Pencil of Rays

Def.

"A pencil of rays is a bundle of rays of light."

There are three kinds of pencils:—

Def.

1. "A divergent pencil is one in which the rays *issue from* a point and form a cone, whose apex is at the point."

e.g. search-light.

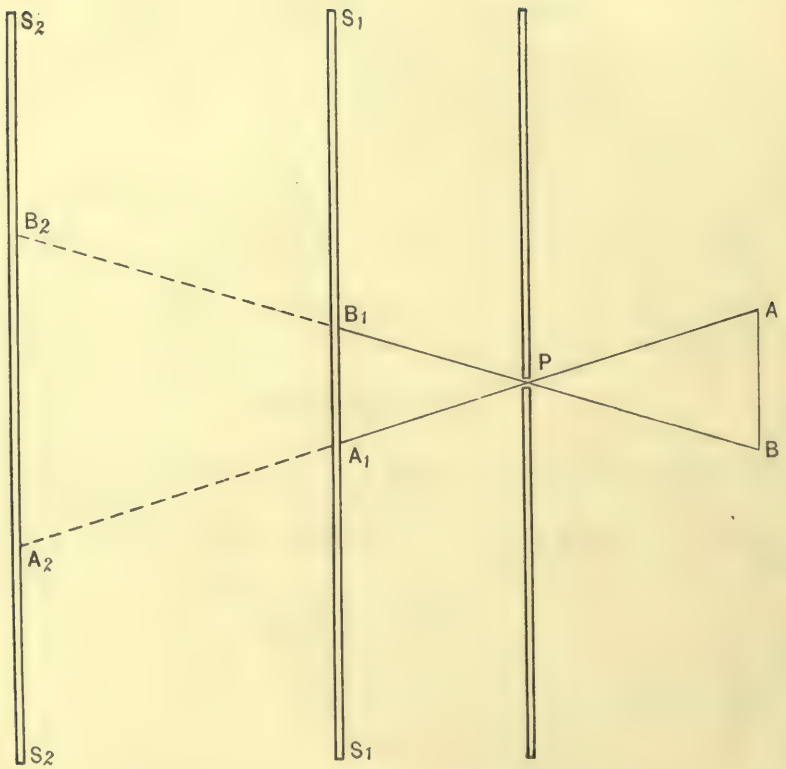
2. "A convergent pencil is one in which the rays *proceed to* a point and form a cone, whose apex is at the point."

e.g. convex lens or burning glass.

3. "A parallel pencil is one in which the rays are parallel."

e.g. the light from a distant object such as the sun.

The light from a luminous object is usually *divergent*.


S2
S1
B2
B1
A
P
A1
B
A2
S2
S1

The Pinhole Camera

Let AB = candle flame.

P = 1st screen with pinhole.

SS = 2nd screen.

We arranged the apparatus as shown in the diagram and, by varying the position of the screen SS, obtained two inverted pictures of the candle A_1B_1, A_2B_2.

The experiment proves that light travels in straight lines, because

1. The picture is inverted.
2. The size of the picture is greater the further we move SS from P.

We may also notice the following points:—

1. If the pinhole were a point, *i.e.* infinitely small, a single ray would pass through the pinhole from each point on the object. These rays cross at the pinhole and so give an inverted image.

2. If the pinhole is *not* infinitely small, each point on the object is the source of a small *divergent* pencil, which, passing through the aperture, forms on the screen a bright image *of the aperture*. If the aperture is round, the image is circular; if triangular, the image is triangular. It is by the blending together of an infinite number of these images that the image of the candle flame is built up.

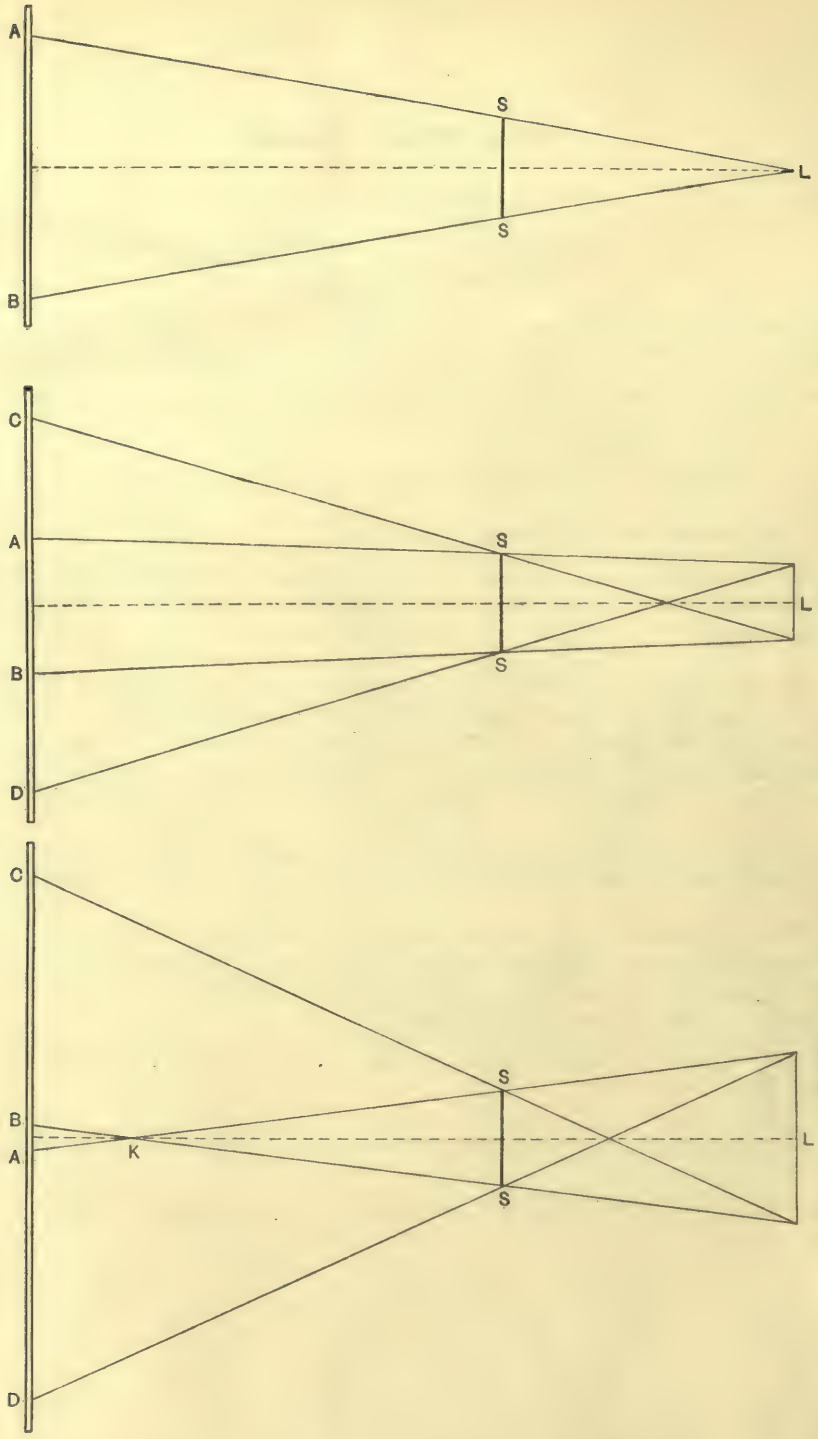
3. If there are two pinholes there will be two images; if a number of holes, or one big hole, a number of images giving a blurred effect, or the effect of light.

4. This experiment explains the formation of the circular patches of light seen under trees in summer. They are images of the sun, an aperture through the leaves acting as a pinhole. When the rays from the sun are not vertical, the circular patch gets drawn out and we get an oval patch.

Proof of this is obtained in an eclipse of the sun, when the patches become *crescent-shaped.*

5. The principle explained here has been used in the making of a photographic camera.

(The word "camera" means a box; cf. "camera obscura," which is a "dark room.")


A
S
L
S
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C
A
S
L
B
S
D
C
S
B
A
K
L
S
D

Shadows

When an opaque body is placed between a source of light and a screen, it cuts off light from the screen, owing to the Rectilinear Propagation of Light, and a shadow of some sort is produced.

The nature of this shadow depends upon the relative size of the source and the opaque body.

We took a fish-tailed burner, a piece of cardboard, 15 by 3 cms., and a screen, and arranged them as shown in the diagrams. We increased the size of the flame gradually and could distinguish between three distinct stages :—

1. When the source of light is a point.

L = source.

SS = opaque body.

AB = umbra.

2. When the source of light is smaller than the opaque body.

AB = umbra.

AC, BD = penumbra.

The penumbra is not all of uniform darkness since, as we go from A to C, we pass gradually from complete darkness to complete light.

3. When the source of light is greater than the opaque body.

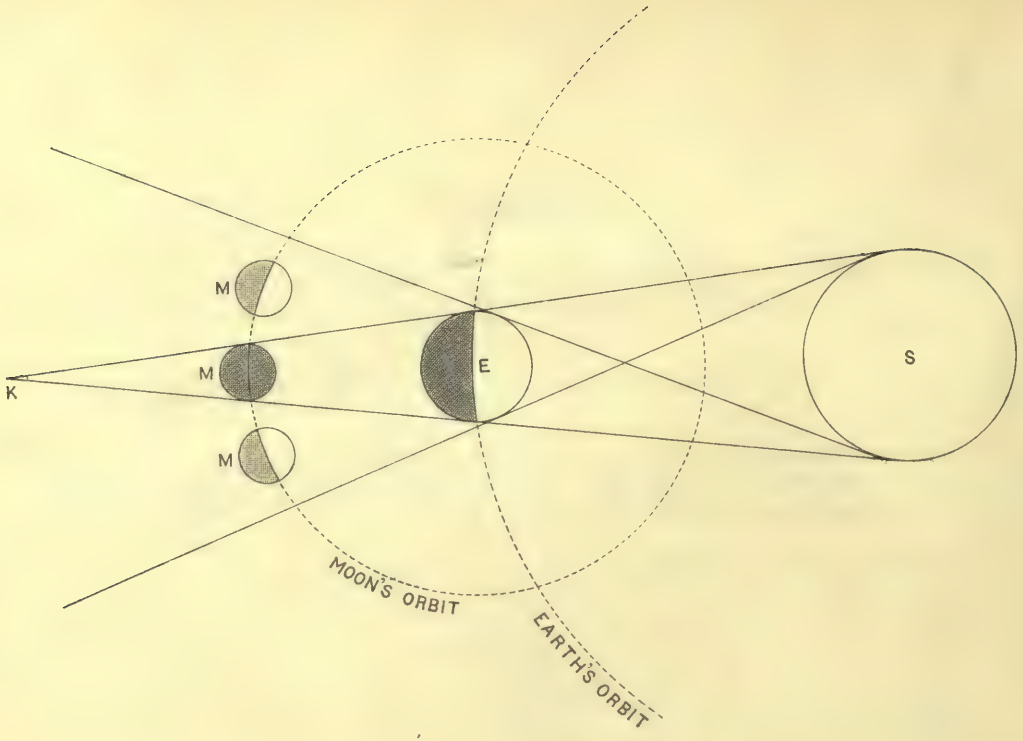
SKS = umbra.

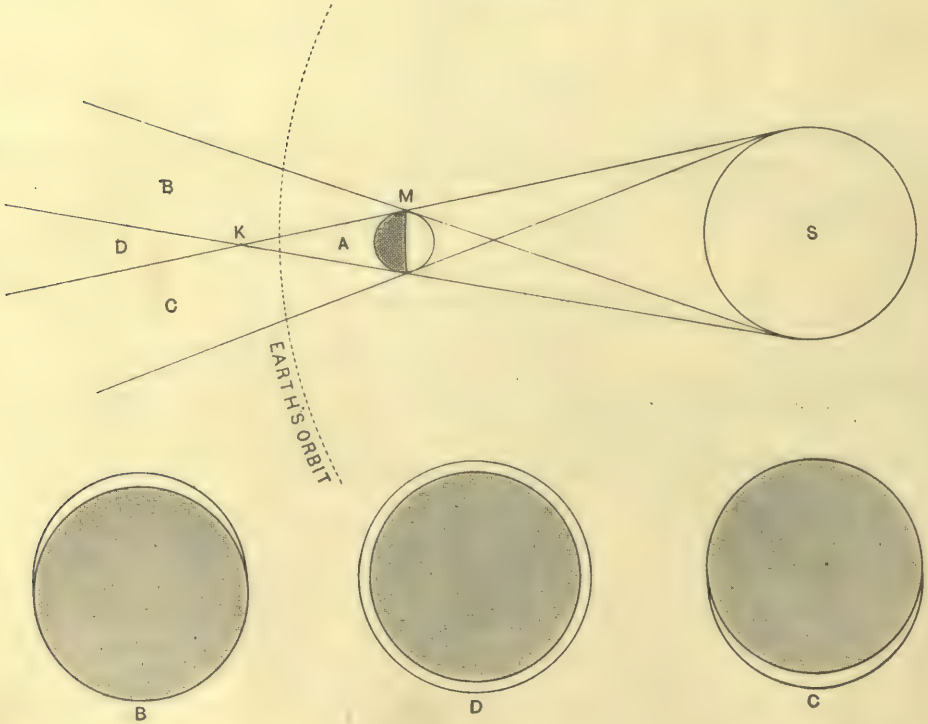
$AKSC$, $BKSD$ = penumbra.

AKB = penumbra of special character.

The area AKB is peculiar, since the light that reaches it at any point comes from both the top and the bottom of the source.

This is important in connection with "annular eclipses."


M
M
M
K
E
S
MOON'S ORBIT
EARTH'S ORBIT


B
M
D
K
A
S
C
EARTH'S ORBIT
B
D
C

Eclipses

The earth and the moon, being opaque spheres, cast shadows behind them of a conical nature by intercepting the sun's rays.

1. If the moon, in its revolution round the earth, passes through the shadow cast by the earth, we get an "eclipse of the moon."

2. If the earth, in its revolution round the sun, passes through the shadow cast by the moon, we get an "eclipse of the sun."

Lunar Eclipse or Eclipse of the Moon

The moon in revolving round the earth enters

1.	Penumbra.	Partial Eclipse.
2.	Umbra.	Total Eclipse.
3.	Penumbra.	Partial Eclipse.

The eclipse begins gradually, the moon's light becoming steadily weaker, till we get totality as it enters the umbra. On leaving the umbra the reverse process takes place.

The reason why a lunar eclipse does not take place every month *at full moon* is because the moon's path round the earth shifts, the moon passing sometimes to one side and sometimes to the other of the earth's shadow.

Solar Eclipse or Eclipse of the Sun

We may distinguish between four kinds of eclipse:—

1. In *A* the eclipse is total.

2. In *B* the upper part of the sun alone is visible, giving a crescent-shaped sun or a partial eclipse.

3. In *C* the lower part of the sun alone is visible and the eclipse is partial.

4. In *D* the eclipse is *annular*, the sun appearing as a ring of light with the dark moon in the centre.

(Annulus = a ring.)

In the diagram *K* is the apex of the cone of umbra and is about 240,000 miles from the moon. Now the distance of the earth from the moon is sometimes less and sometimes more than 240,000 miles. An annular eclipse is obtained when the distance is *more* than this amount.

PHOTOMETRY

Photometry is the measurement of light. Different sources of light have different "illuminating powers."

Standards of Illuminating Power

To compare illuminating powers we must have some standard or unit. The unit usually employed is the illuminating power of the "standard candle."

Def.

"Standard candles are sperm candles, $\frac{7}{8}$ in. diameter, 6 to the lb., burning at the rate of 120 grains per hour."

The illuminating power of a source is defined in terms of the standard candle.

Def.

"The illuminating power or candle power of a source is the number of standard candles, which will give the same illumination at all points, if put in the same place as the source."

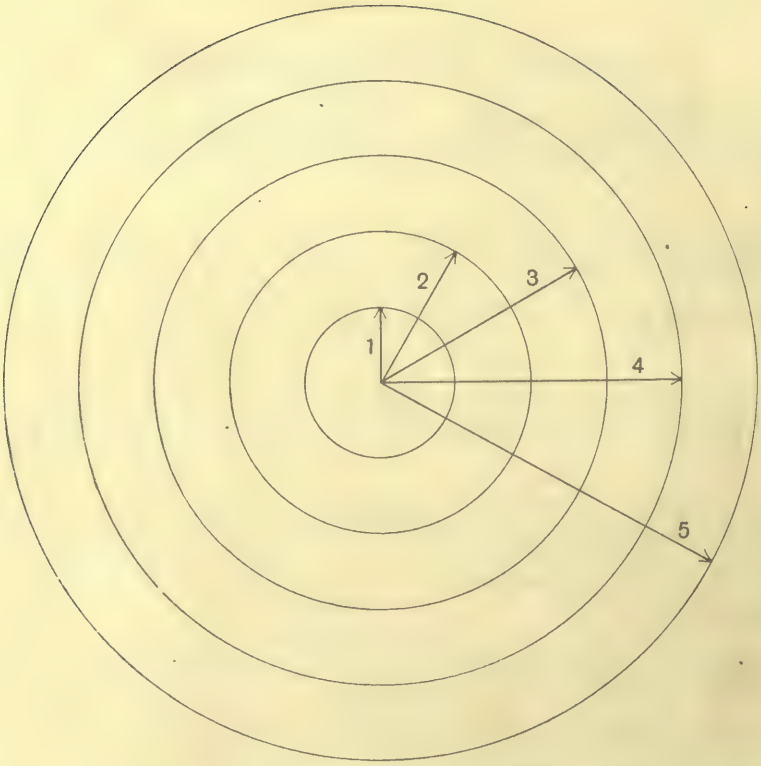
It has been found, however, that, as a standard of illuminating power, the standard candle is unsuitable and unreliable.

A number of alternative standards have been suggested, of which the following are the most important:—

1. The Pentane standard lamp, invented by Vernon Harcourt, which burns normal pentane. It is the standard used in Great Britain.

2. The Carcel standard lamp, invented by Regnault and Dumas, which burns colza oil. It is the standard used in France.

3. The Amyl Acetate standard lamp, invented by Hefner-Alteneck, which burns amyl acetate. It is the standard used in Germany.


1
2
3
4
5

Standards of Illumination

It is important to distinguish carefully between "illumination" and "illuminating power."

Def.

"The illumination at a point due to a given source is the brightness of a surface placed perpendicularly to the line joining the point to the source."

Strictly speaking, we should use the term "intensity of illumination at a point" for "illumination at a point."

To compare the illumination at various points we must have some standard or unit The unit usually employed is the candle-centimetre or candle-foot.

Def.

"The candle-cm. is the illumination at a point due to 1 candle at a distance of 1 cm."

"The candle-ft. is the illumination at a point due to 1 candle at a distance of 1 ft."

The comparison of the illuminating powers of different sources is made on the assumption that two candles give at all points twice the illumination of one.

Law of Inverse Squares

Def.

"The illumination at a point due to a given source is inversely proportional to the square of the distance of the point from the source."

The proof of this law depends upon the assumption that if light, which is spread over any area, is then spread over an area ten times as great, the illumination at a point will be one-tenth the value.

Proof of the Law of Inverse Squares

Area of sphere $= 4\pi r^2$.

i.e. the area of a sphere is directly proportional to the square of its radius.

Let us consider a source radiating out light in all directions and find the illumination at a point on imaginary spheres *surrounding it*, placed at different distances from the source, which is in the centre.

Radius of sphere	Area of sphere	Illumination at a point	(Distance)^2	Test
1	1	1	1	1
2	4	$\frac{1}{4}$	4	1
3	9	$\frac{1}{9}$	9	1
4	16	$\frac{1}{16}$	16	1
5	25	$\frac{1}{25}$	25	1

The last column is a test of inverse proportion, and proves that the illumination at a point due to a given source is inversely proportional to the square of the distance of the point from the source.

This is the Law of Inverse Squares and it is true in all cases.

Application of the Law of Inverse Squares

1. If the illumination at a point due to a given source is 2000 candle-cms. at a distance of 1 cm., what will it be at a distance of 25 cms.?

$$\text{Illumination due to source at 1 cm.} = 2000 \text{ candle-cms.}$$

$$\text{Illumination due to source at 25 cms.} = \frac{2000}{25^2}$$

$$= \frac{2000}{625}$$

$$= \frac{80}{25}$$

$$= \frac{16}{5}$$

$$= 3{\cdot}2 \text{ candle-cms.}$$

$$\text{Illumination} = 3{\cdot}2 \text{ candle-cms.}$$

2. Find the illumination at a point 10 cms. from a source of 250 candle power.

$$\text{Illumination due to 250 candles at 1 cm.} = 250 \text{ candle-cms.}$$

$$\text{Illumination due to 250 candles at 10 cms.} = \frac{250}{10^2}$$

$$= \frac{250}{100}$$

$$= 2{\cdot}5 \text{ candle-cms.}$$

$$\text{Illumination} = 2{\cdot}5 \text{ candle-cms.}$$

3. Find the candle power of a source which gives the same illumination at 25 ft. as a standard candle at 5 ft.

Let x = candle power of the source.

$$\text{Illumination due to 1 candle at 5 ft.} = \frac{1}{5^2} \text{ candle-cms.}$$

$$\text{Illumination due to } x \text{ candles at 1 ft.} = x \text{ candle-cms.}$$

$$\text{Illumination due to } x \text{ candles at 25 ft.} = \frac{x}{25^2} \text{ candle-cms.}$$

Since the illumination due to the two sources is the same,

$$\frac{x}{25^2} = \frac{1}{5^2}.$$

$$\therefore x = \frac{25^2}{5^2}$$

$$= 5^2$$

$$= 25.$$

$$\text{Candle power of source} = 25.$$

S
C
D
R
A
B
S

Photometers

A Photometer is an instrument for measuring the illuminating power or candle power of a source.

I. Rumford's Photometer

We took an apparatus consisting of the following parts:—

SS = screen.
R = rod of retort-stand.
A and B = two sources.

We arranged them as shown in the diagram and varied the positions of A and B until the two shadows, C and D, cast upon the screen, were equally illuminated.

Note that the source A illuminates the shadow C, and the source B the shadow D, *i.e.* the source A at the distance AC gives the same illumination as the source B at the distance BD.

To find the candle power of a source

We took a standard candle and an 8 C.P. lamp, and arranged them so that the two shadows were of equal intensity. We measured the distances AC and BD.

Distance of candle from screen = 40 cms.
Distance of lamp from screen = 129 cms.

Let x = candle power of the lamp.

Illumination due to 1 candle at 40 cms. $= \frac{1}{40^2}$ candle-cms.

Illumination due to x candles at 129 cms. $= \frac{x}{129^2}$ candle-cms.

Since the illumination due to the two sources is the same,

$$\frac{x}{129^2} = \frac{1}{40^2}.$$

$$\therefore x = \frac{129^2}{40^2}$$

$$= \frac{16641}{1600}$$

$$= \frac{166{\cdot}41}{16}$$

$$= 10{\cdot}40.$$

Candle power of lamp = 10·40.

Practical points to notice

1. Place the retort-stand near to the screen. This gets rid of penumbra and gives a sharp shadow.

2. Move up the lamp from one direction and then from the other, keeping the candle fixed, and take the mean of the readings.

3. Make at least two sets of observations with the candle at different distances and *work them out separately.*

Objection to Rumford's Photometer

The tone of the two shadows is often very different and it is difficult to decide when the intensity is equal.

II. Bunsen's Photometer

We took a piece of white paper with a grease spot on it and viewed it by transmitted and reflected light.

1. Transmitted light.

The grease spot appears *light on a dark ground,* since it transmits more light than the rest of the paper.

2. Reflected light.

The grease spot appears *dark on a light ground,* since it reflects less light than the rest of the paper.

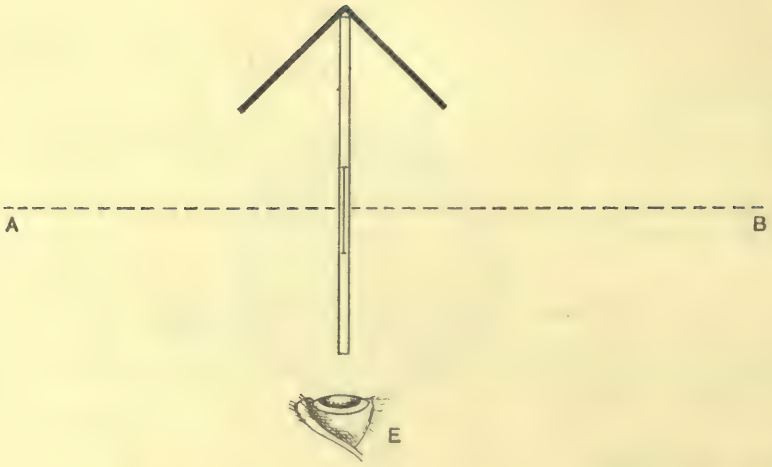
If the paper is turned at a certain angle to the light, the grease spot is seen to disappear.

The Bunsen photometer or Bunsen disc used to consist of a piece of paper with a grease spot in the middle. It is now made of a piece of greased paper with a painted ring on it. The ring disappears if the disc is turned at a certain angle to the light. To find out when this occurs, we made the following experiment.

To test a Bunsen Photometer

We took a Bunsen photometer and arranged a standard candle on either side of it. We altered the position of the photometer until the ring disappeared and measured the distances of the candles from the photometer. The distances were practically the same, which means that the illumination on both sides of the photometer was the same.

We concluded from this that *the ring disappears when the illumination on the two sides is the same.*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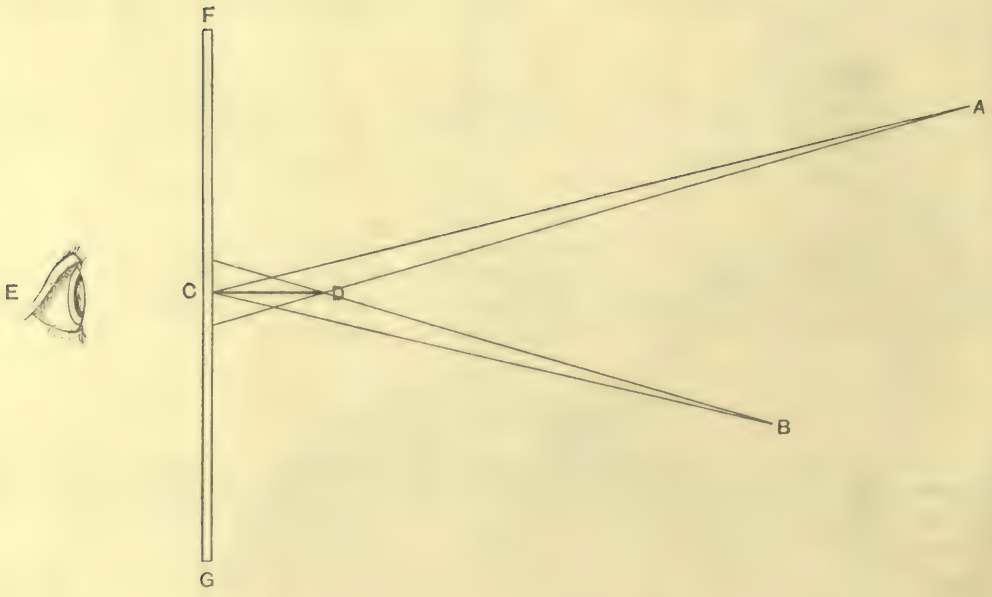
F
A
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C
D
B
G

To find the candle power of a source

We arranged two sources, a standard candle and an 8 c.p. lamp, one on either side of the photometer, and varied the position of the lamp until the ring disappeared. We measured the distance of each source from the photometer.

Distance of candle from photometer = 25 cms.

Distance of lamp from photometer = 78 cms.

Let x = candle power of the lamp.

Illumination due to 1 candle at 25 cms. $= \frac{1}{25^2}$ candle-cms.

Illumination due to x candles at 78 cms. $= \frac{x}{78^2}$ candle-cms.

Since the illumination due to the two sources is the same,

$$\frac{x}{78^2} = \frac{1}{25^2}.$$

$$\therefore \quad x = \frac{78^2}{25^2}$$

$$= \frac{6084}{625}$$

$$= 9{\cdot}73.$$

Candle power of lamp = 9·73.

Practical points to notice

1. View the photometer *from the side* and not from behind the source of light.

2. Move up the lamp from one direction and then from the other, keeping the candle fixed, and take the mean of the readings.

3. Since the ring disappears in different positions, according as it is viewed from one side or the other, take readings for both sides and find the mean of the results.

Bunsen's Photometer with Mirrors

In this instrument the two sides of the photometer are seen simultaneously by having two mirrors, one on either side, inclined at an angle of 45°.

This adds greatly to the sensitiveness of the instrument.

III. Bouguer's Photometer

This is an improved form of Rumford's Photometer.

FG = translucent screen of ground glass.

CD = blackened opaque screen.

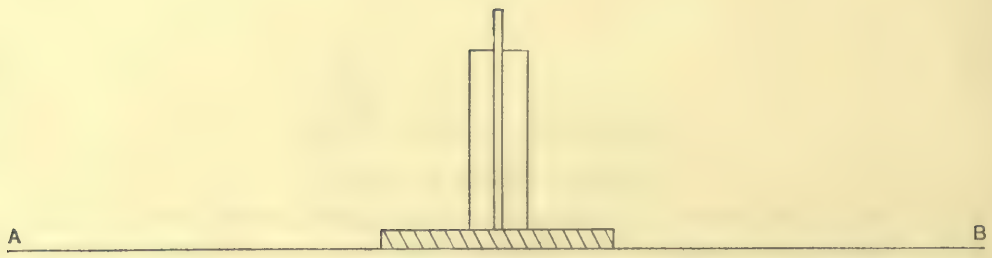
A and B = two sources.

E = eye.

Two shadows are cast on FG, which are illuminated by A and B. By varying the positions of A and B, the shadows may be obtained of the same intensity. They are viewed from E.

Advantage—Absence of head.

Disadvantage—The two shadows are separated at C.


A
B

IV. Joly's Photometer

A screen is formed from two similar parallel slabs of paraffin wax, placed face to face, with a sheet of polished tinfoil interposed between them. The screen is placed between the sources, so that each is illuminated only by one source. The light is scattered and the slabs, which appear bright, are viewed sideways. The sources are adjusted till the two slabs appear equally bright.

Law of Direct Squares

This must not be confused with the Law of Inverse Squares.

Def.

"The candle powers of two sources are directly proportional to the squares of the distances at which they give the same illumination."

Proof of the Law of Direct Squares

Let x = candle power of a source A,
y = candle power of a source B,

and let A at m cms. give the same illumination as B at n cms.

$$\text{Illumination due to } x \text{ candles at } m \text{ cms.} = \frac{x}{m^2},$$

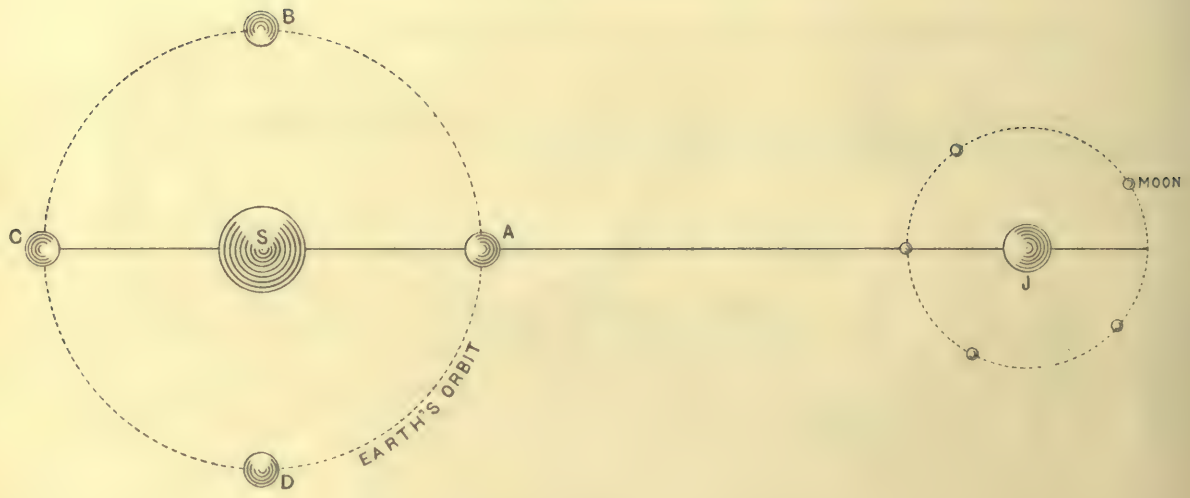
$$\text{Illumination due to } y \text{ candles at } n \text{ cms.} = \frac{y}{n^2}.$$

Since the illumination due to the two sources is the same,

$$\frac{x}{m^2} = \frac{y}{n^2}.$$

$$\therefore \frac{x}{y} = \frac{m^2}{n^2}.$$

i.e. the candle powers are *directly* proportional to the squares of the distances at which they give the same illumination.


B
C
S
A
MOON
J
D
EARTH'S ORBIT

VELOCITY OF LIGHT

It is a familiar fact that, when a gun is fired, we see the flash before we hear the report. This shows that light travels faster than sound.

Velocity of sound = 1100 ft. per sec.

Velocity of light = 186,000 miles per sec.

The determination of the velocity of sound is therefore comparatively easy, whilst that of light is a far more difficult matter.

The first attempt to determine the velocity of light was made by Galileo. He stationed two observers, A and B, at a considerable distance apart, with lamps, which could be covered up. A uncovered his lamp and B did the same as soon as possible after seeing the light from A's lamp. The interval measured by A gave the time taken by the light to go from A to B and back again. It was impossible to eliminate the "personal equation," and in any case the experiment could not lead to any trustworthy results.

This method of Galileo is really the one employed by Fizeau on a much more elaborate scale.

Determination of the Velocity of Light

I. Römer's Method

In 1676 Römer, a Dane, was making observations upon the moons of Jupiter, which are five in number. He found that in each revolution the moons plunged into the shadow cast by the sun, thus causing periodic eclipses. *He determined the interval between two successive eclipses of a particular moon*, when the earth was at A, and found it to be 42 hours 28 mins. 35 secs. He calculated the time at which eclipses should take place during the coming year, but he found that, as the earth moved from A to B, the eclipses occurred behind the calculated times and the amount of the retardation increased until, after six months, it reached a maximum when the earth was at C, the eclipse taking place 16 mins. 36 secs. late. As the earth returned through D to A, the eclipses took place more nearly at the calculated times until, at the end of a year, when the earth was once more at A, the observed and calculated times coincided.

Römer argued that the retardation of 16 mins. 36 secs. was due to the time taken by the light in travelling from A to C, *i.e.* across the diameter of the earth's orbit.

Distance of earth from sun = $92\frac{1}{2}$ million miles.

Diameter of earth's orbit = 185,000,000 miles.

Time = 16 mins. 36 secs.

= 996 secs.

In 996 secs. light travels 185,000,000 miles.

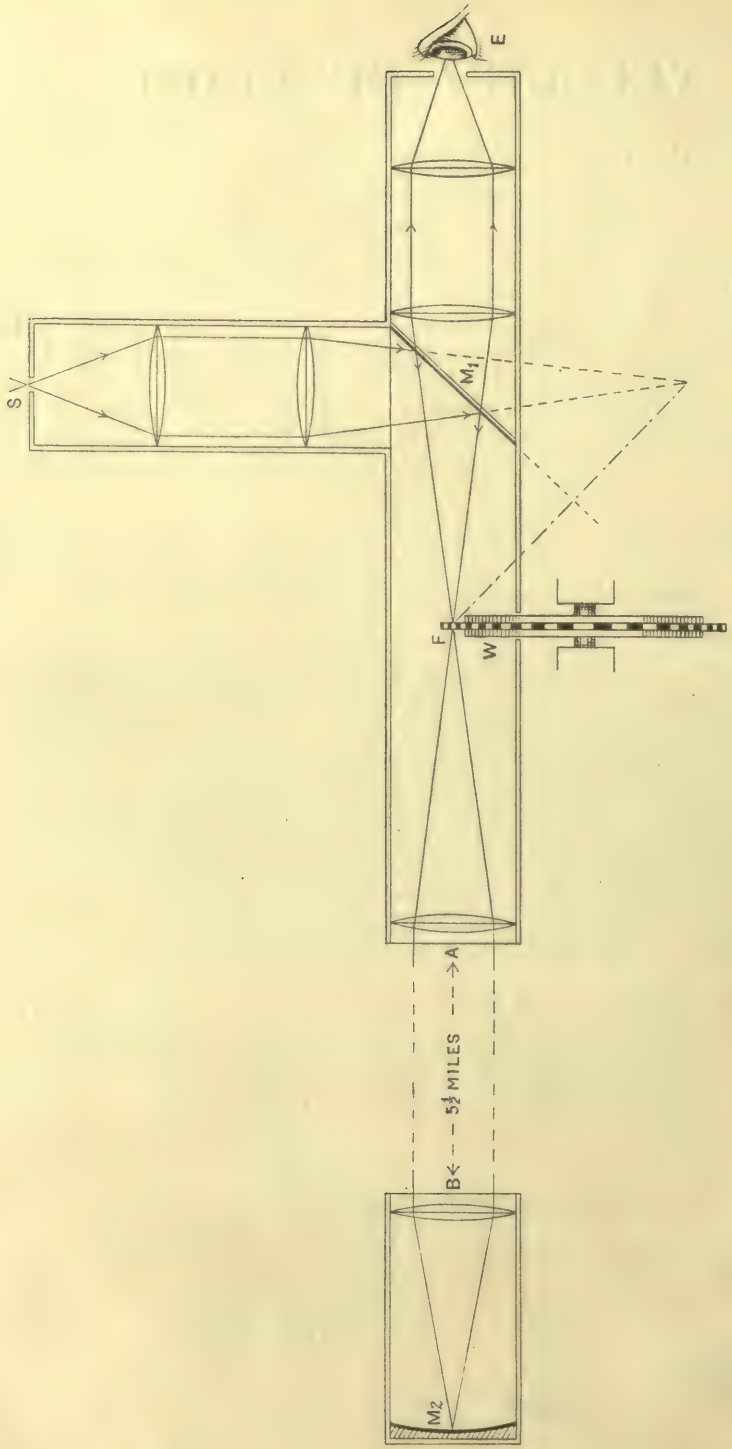
In 1 sec. light travels $\frac{185,000,000}{996}$

= 186,000 miles.

Velocity of light = 186,000 miles per sec.

II. Bradley's Method

(See advanced notes.)


E
S
M1
F
W
A
5½ MILES
B
M2

III. Fizeau's Method

Let

S = source of Light.

M_1 = plane unsilvered mirror.

W = toothed wheel.

F = top of wheel.

A—B = distance of $5\frac{1}{2}$ miles.

M_2 = concave mirror.

E = eye of observer.

Light from a source S, after passing through two lenses, is reflected at the mirror M_1 and brought to a focus at F. If the wheel is in such a position that there is a space at F, the light will diverge to the lens at A. Being rendered parallel by this lens, the rays pass to another lens at B, by means of which they are brought to a focus at M_2. Here they once more undergo reflection and travel back along the same course, except that, on reaching M_1, some of the rays pass through the mirror to the eye at E.

If the wheel is at rest, with a space at F, an observer at E will see continuous light, but, if the wheel is rotated slowly, some of the light which passes through a space will be cut off on returning by the next tooth. As the rate of the wheel is increased, more and more light will be cut off until eventually there is *total eclipse*. It is plain, when this is the case, that, in the time taken for a tooth to replace a space, the light has travelled from F to M_2 and back again. The time taken to replace a space by a tooth can be found by knowing the time of revolution of the wheel and the number of the teeth.

Disadvantages of Fizeau's Method

1. The light from S is greatly weakened on its return by reflection at M_1, so that the image seen at E is very faint.

2. The light from S, when intercepted by a tooth, is reflected back towards the eye, so that there will be a certain amount of general illumination. This can be partly avoided by bevelling the teeth, so that the light is reflected towards the sides of the tube.

Example

In one of Fizeau's experiments the wheel had 710 teeth and revolved 12 times per sec. The distance of the wheel from the far mirror was $5\frac{1}{2}$ miles. Find the velocity of light.

Number of spaces and teeth $= 710 \times 2$

$= 1420.$

Time of 1 revolution $= \frac{1}{12}$ sec.

$\therefore$ Time of $\frac{1}{1420}$ revolution $= \frac{1}{12 \times 1420}$

$= \frac{1}{17{,}040}$ sec.

Distance light has travelled $= 5\frac{1}{2} \times 2$

$= 11$ miles.

In $\frac{1}{17{,}040}$ sec. light travels 11 miles.

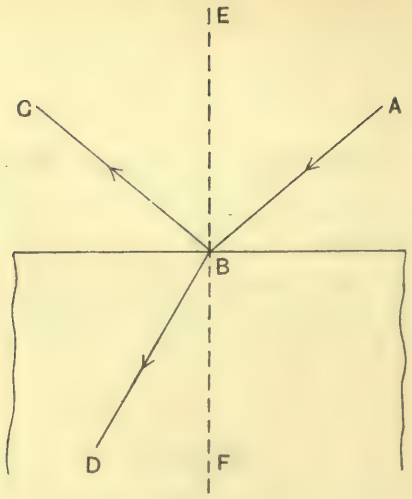
In 1 sec. light travels $11 \times 17{,}040$

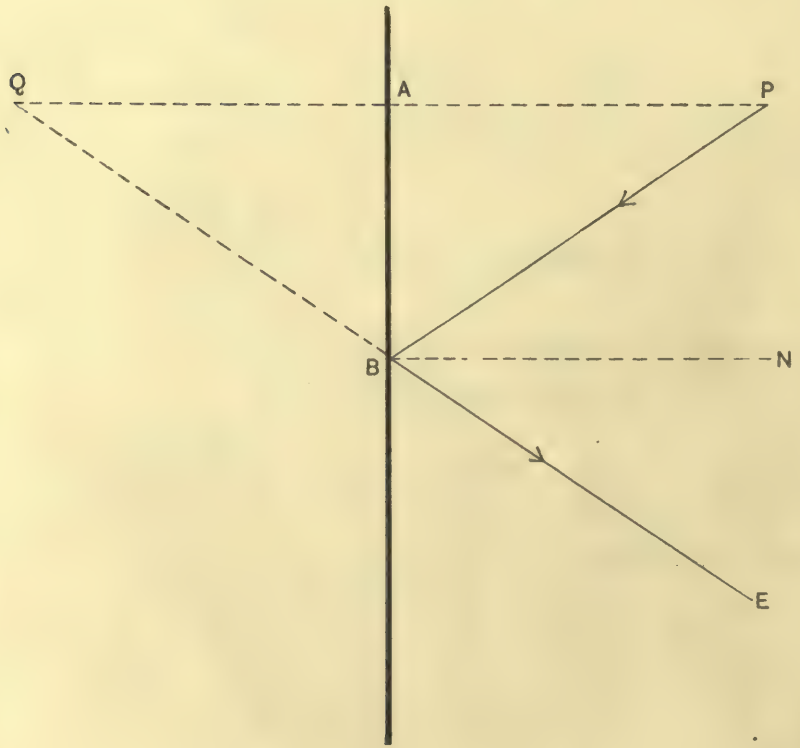
$= 187{,}440$ miles.

Velocity of light $= 187{,}440$ miles per sec.

IV. Foucault's Method

(See advanced notes.)


E
C
A
B
D
F


Q
A
P
B
N
E

REFLECTION OF LIGHT

When a ray of light falls on a polished surface bounding two transparent media, part of the light is transmitted and part reflected.

That this is so is plain, if we look at a polished glass surface, since we obtain reflections of objects in it, which are called "images."

To find the position of the image formed by reflection at a plane surface

We took a burner P and a plain sheet of glass, and found the position of the image of P seen by reflection in the glass plate by placing another burner Q in such a position that Q and the image of P coincided in all positions of the eye.

Let P = object.
Q = image.
E = position of eye.

We found by experiment that

1. $AP = AQ$.
2. AB, PQ are at right angles to one another.

We drew EBQ, dotting the part BQ, which is behind the surface, and joined PB. We drew BN perpendicular to the surface at B, dotting it.

The diagram shows various things:—

1. Normal to surface, BN.

Def.

"The normal to a surface at a point is the line drawn perpendicularly to the surface at that point."

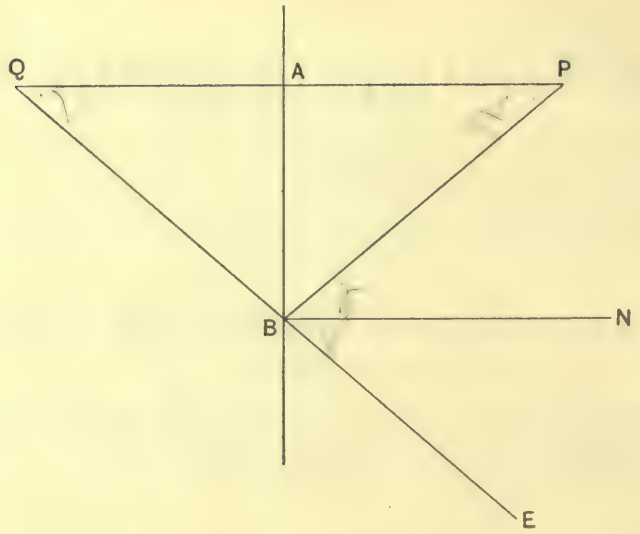
2. Incident ray, PB.
3. Reflected ray, BE.
4. Angle of Incidence, PBN.
5. Angle of Reflection, EBN.

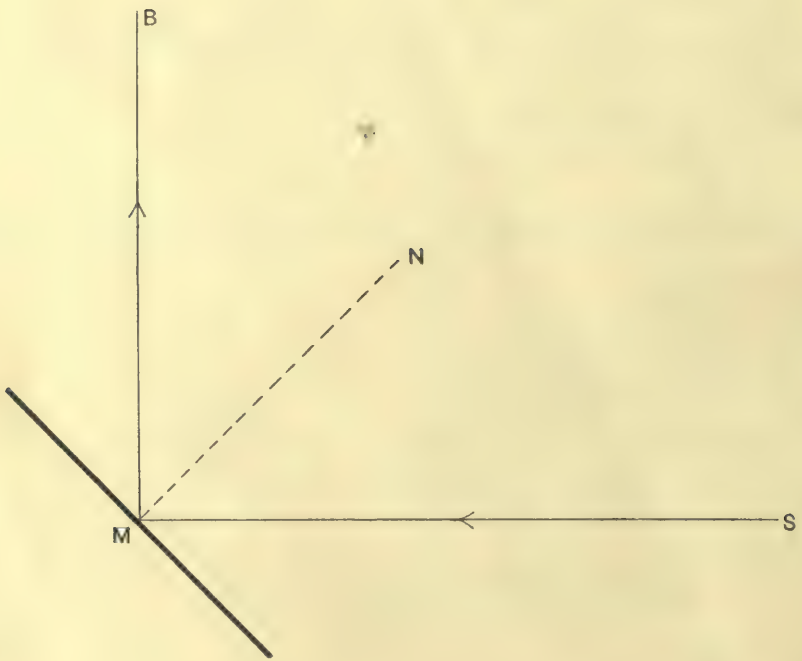
We measured with a protractor the angles of incidence and reflection.

Angle of incidence = 34·0°.
Angle of reflection = 34·5°.

This shows that

Angle of incidence = Angle of reflection.


Q
A
P
B
N
E


B
N
M
S

To prove geometrically that

angle of incidence = angle of reflection

We have learnt by experiment the following facts:—

1. $AP = AQ$.
2. Angles PAB, QAB are right angles.

In the triangles PAB, QAB,

$AP = AQ$.
AB is common.
$\angle PAB = \angle QAB$.

$\therefore$ The triangles PAB, QAB are congruent.

$\therefore \angle APB = \angle AQB$.

But since QP and BN are parallel,

$\angle APB = \angle PBN$,
$\angle AQB = \angle EBN$.
$\therefore \angle PBN = \angle EBN$.

i.e. Angle of incidence = Angle of reflection.

Laws of Reflection

Def.

I. "The incident ray, the normal to the surface and the reflected ray lie in one plane."

II. "The incident and reflected rays make equal angles with the normal to the surface."

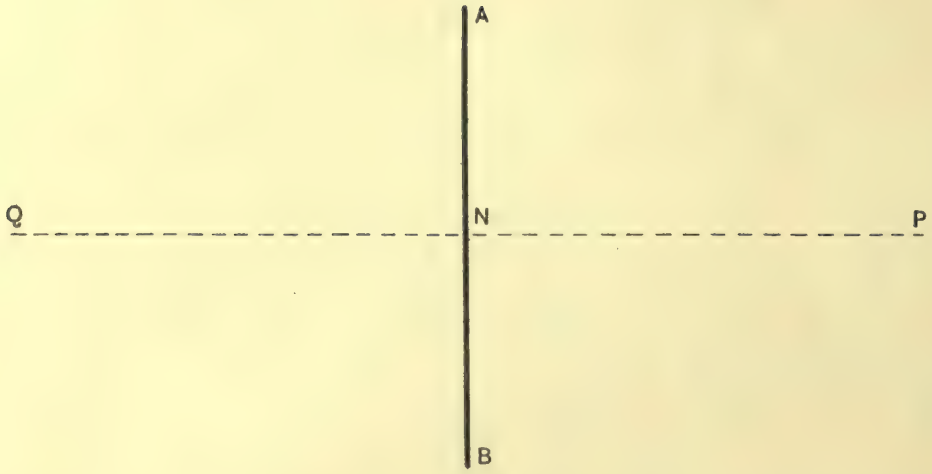
Experimental Proof of the Laws of Reflection

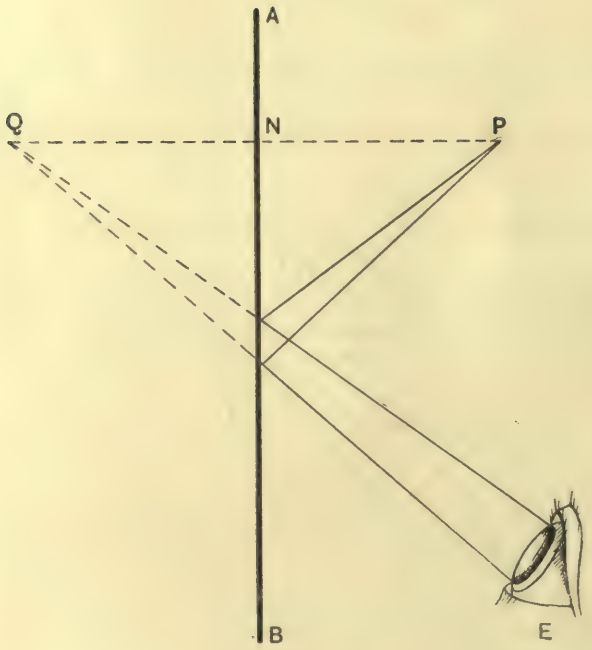
We took a lantern and arranged it so as to give a parallel beam of light, and fixed a mirror in such a position that the beam was reflected on to the ceiling. The mirror had a piece of wood attached at right angles to the surface to act as a normal to the surface. We measured the angles which the incident and reflected rays made with the normal.

Let
S = source giving out parallel beam.
M = mirror.
MN = normal to the surface.
SM = incident ray.
MB = reflected ray.

By this experiment we can show that

1. SM, MN and MB lie in one plane.
2. $\angle SMN = \angle BMN$.


A
Q
N
P
B


A
Q
N
P
B
E

Virtual and Real Images

Def. Virtual Image.

"When a pencil of rays, diverging from a point, after reflection or refraction, *appears to diverge from* a second point, the second point is called the virtual image of the first point."

Virtual images are produced by

1. Plane mirror.
2. Spherical mirror.
3. Lens.

Def. Real Image.

"When a pencil of rays, diverging from a point, after reflection or refraction, *converges to* a second point, the second point is called the real image of the first point."

Real images are produced by

1. Spherical mirror.
2. Lens.

In the case of a virtual image the rays which form it do not actually pass through it. In the case of a real image the rays which form it actually pass through it and the image can be obtained on a piece of paper.

To find the image of a point formed by reflection in a plane mirror

Def.

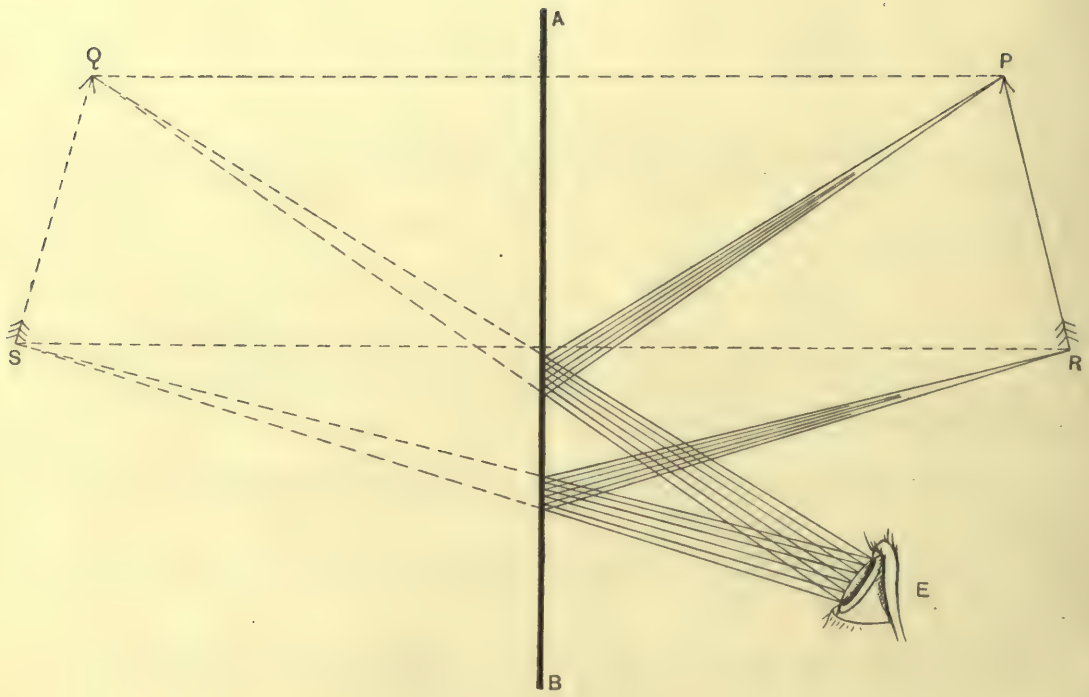
"Draw a normal from the point to the mirror and produce it as far behind the mirror as the point is in front."

To draw the rays by which an eye sees a luminous point by reflection in a plane mirror

Let AB = plane mirror.
P = luminous point.
E = eye.

We found the image Q by the method just given. We then drew, with E as base, a cone of rays to Q, dotting them where they do not exist. We joined up the points where the rays cut the mirror to P.

N.B.—In all optical diagrams it is essential to find the image before drawing the rays by which the eye sees it.


A
Q
P
S
R
E
B

To draw the rays by which an eye sees a luminous object by reflection in a plane mirror

Let PR = object.

We found the images of two points on it, P and R, which are Q and S, by the usual method and drew the rays by which the eye sees these points.

We see from this that a plane mirror produces a virtual image of an object which is the same size as the object.

Lateral Inversion

The image in a plane mirror is *laterally inverted*, *i.e.* right becomes left and left becomes right.

e.g. Parting of the hair.
Writing backwards.

In the case of a mirror or lens we often get an *up-and-down inversion*, the image being upside down.

Reflecting Power

The reflecting power of a surface depends upon

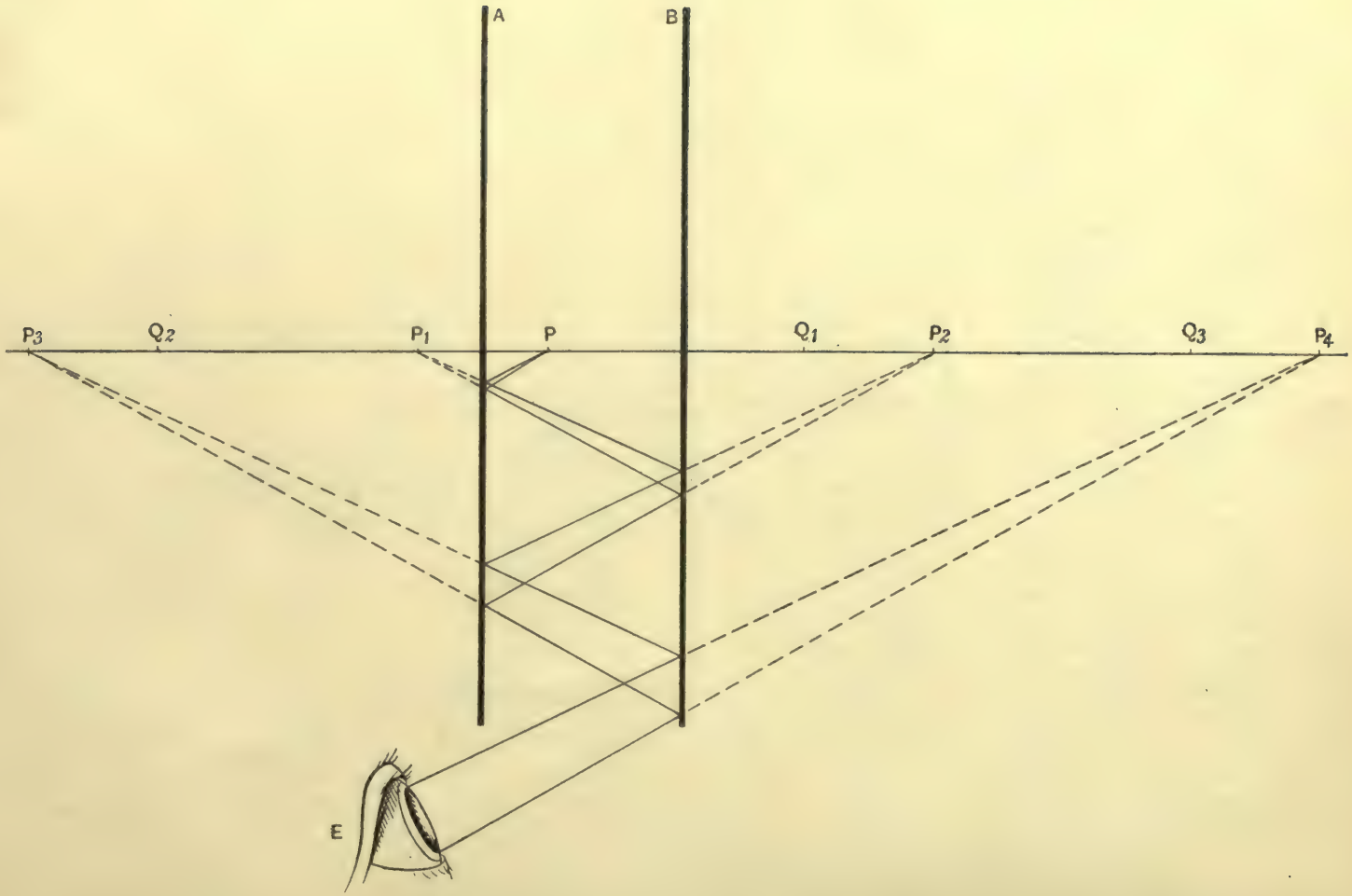
1. Material of the surface.
2. Angle of incidence.

1. If the light is perpendicular

Material	% reflected
Water	1·8
Glass	2·5
Mercury	66·6

2. For water

Angle	% reflected
40°	2·2
60°	6·5
80°	33·5
89·5°	72·0

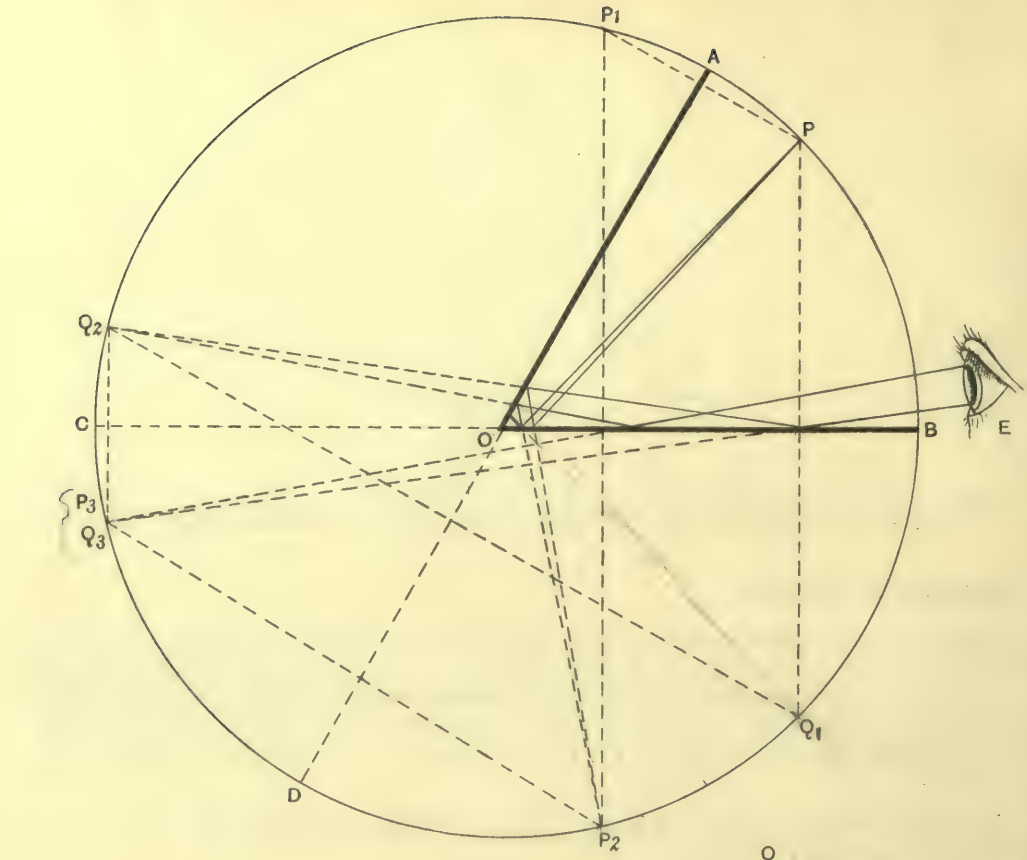

A
B
P3
Q2
P1
P
Q1
P2
Q3
P4
E

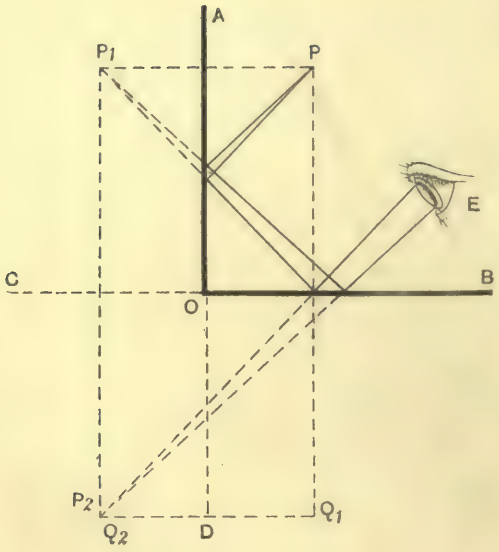
Images formed by the Combination of Two Plane Mirrors

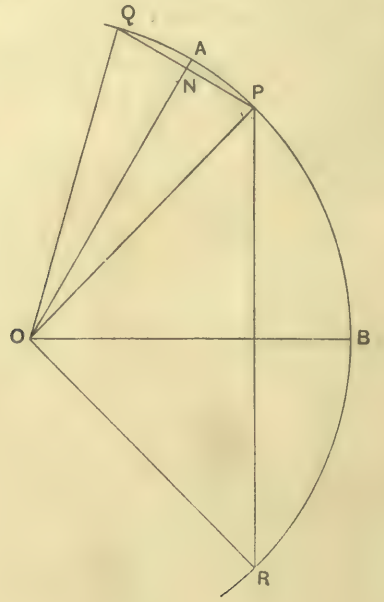
I. Two Parallel Mirrors

We placed two mirrors parallel to one another and put an object between them. One of the mirrors had a piece of silvering removed at one point and, looking through this, we saw an *infinite number* of images. There are two different sets of images, P_1, P_2, P_3, etc., and Q_1, Q_2, Q_3, etc. We also drew the rays by which an eye sees P_4, by drawing a cone of rays to P_4, P_3, P_2, P_1, and finally to P, dotting them where they do not exist.

An image is always obtained when an object or image is *in front of* a mirror, but never when it is *behind*.


P1
A
P
Q2
C
O
B
E
P3
Q3
Q1
D
P2


A
P1
P
E
C
O
B
P2
Q2
D
Q1


Q
A
N
P
O
B
R

II. Two Mirrors at an angle of 90°

With the usual construction we obtained *three* images, P_1, Q_1 and P_2 or Q_2, since these two images coincide. P_2 is the image of P_1 in OB, and Q_2 is the image of Q_1 in OA.

III. Two Mirrors at an angle less than 90°

We drew two mirrors at an angle of 60° and placed a point P between them. We obtained the images by the usual method and found that there were *five*, since P_3 and Q_3 coincide.

We noticed that the area COD was behind both the mirrors. It is for this reason that P_3 and Q_3 give no further images.

Taking O as centre and OP as radius we described a circle and found that it passed through the various images.

To prove geometrically that the images lie on the circumference of a circle

In the triangles ONQ, ONP,

$$NQ = NP.$$

ON is common.

$$\angle ONQ = \angle ONP.$$

$\therefore$ The triangles ONQ, ONP are congruent.

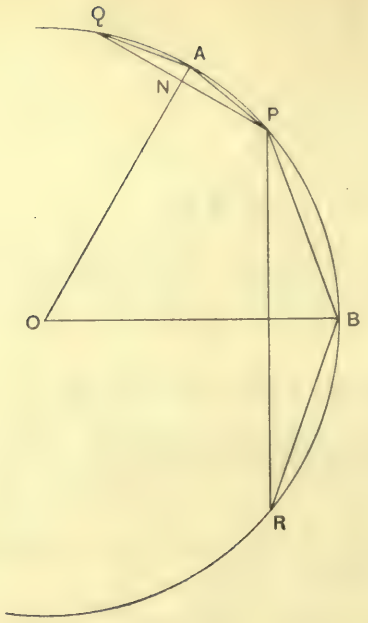
$$\therefore \quad OQ = OP.$$

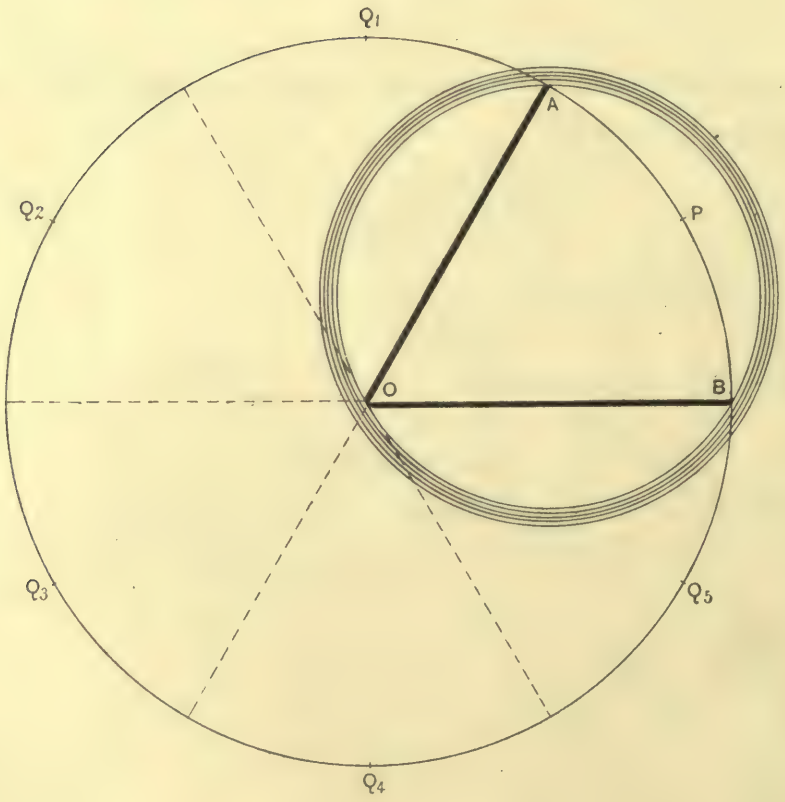
In the same way it can be shown that

$$OR = OP.$$

$$\therefore \quad OQ = OP = OR.$$

i.e. Q, P and R lie on the circumference of a circle with centre O and radius equal to OP


Q
A
N
P
O
B
R


Q1
A
Q2
P
O
B
Q3
Q5
Q4

To obtain a simpler method of finding the Images

We found the images, Q and R, of a point P between two mirrors by describing a circle with centre O and radius OP, and drawing the normals in the usual way.

Let A and B be the points at which the circle cuts the mirrors. Join AP, AQ and BP, BR.

Then in the triangles ANP, ANQ,

$NP = NQ$.

NA is common.

$\angle ANP = \angle ANQ$.

$\therefore$ The triangles ANP, ANQ are congruent.

$\therefore\ AP = AQ$.

In the same way it can be shown that $BP = BR$.

Therefore, to find the image in any mirror, take A or B as centre and cut off equal *chords*.

To find by calculation the number of Images

Let $$n = \frac{180^\circ}{\text{angle}}.$$

Then, if the angle is a sub-multiple of 180°, we can get the number of images as follows :—

$$\text{No. of images} = 2n - 1.$$

e.g. Let angle $= 30^\circ$.

$$\text{Then } n = \frac{180^\circ}{\text{angle}}$$

$$= \frac{180^\circ}{30^\circ}$$

$$= 6.$$

$$\text{No. of images} = 2n - 1$$

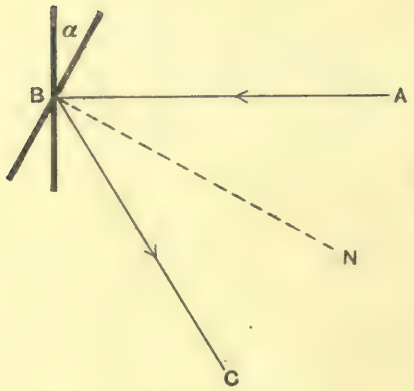
$$= 12 - 1$$

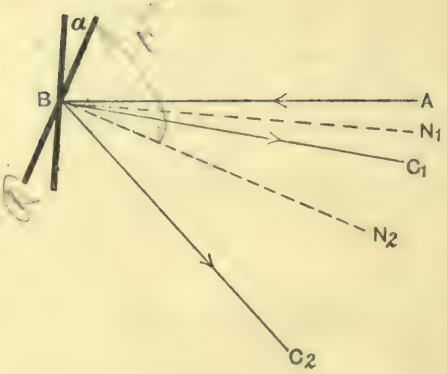
$$= 11.$$

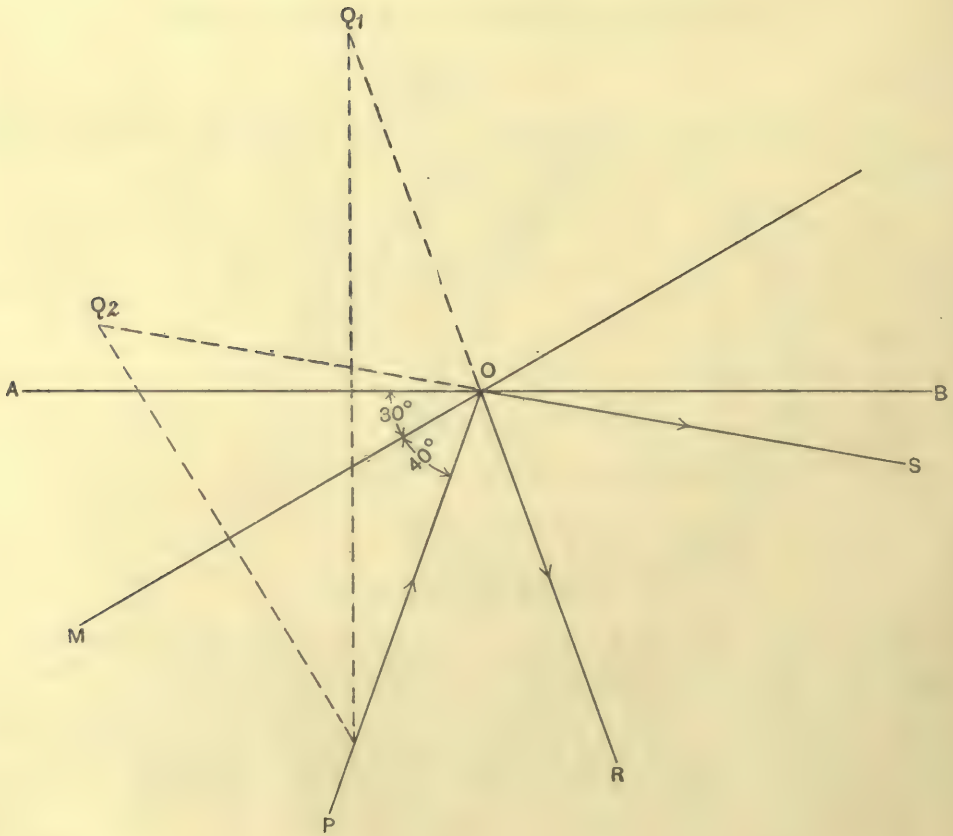
$$\text{No. of images at } 30^\circ = 11.$$

The Kaleidoscope

This consists of a metal tube containing two mirrors extending along its whole length and inclined at an angle of 60° to one another. One end of the tube is closed by a metal plate, which has a hole in the middle. At the other end are two glass plates, an outer one of ground glass and an inner one of clear glass, with a number of pieces of coloured glass lying loosely between them. By turning the end of the tube, symmetrical patterns of various kinds are produced. There are six pictures in all—one object and five images. The instrument is used by designers.


α
B
A
N
C


α
B
A
N1
C1
N2
C2


Q1
Q2
A
O
B
30°
40°
S
M
P
R

To prove that, when a mirror is turned through any angle, the reflected ray is turned through twice that angle

I. Theoretical

1. When the incident ray is perpendicular to the mirror.

Let a = angle through which mirror is turned.

Angle through which reflected ray is turned $= ABC$

$= 2ABN$

$= 2a.$

2. When the incident ray is *not* perpendicular to the mirror.

Angle through which reflected ray is turned $= C_1BC_2$

$= ABC_2 - ABC_1$

$= 2ABN_2 - 2ABN_1$

$= 2(ABN_2 - ABN_1)$

$= 2N_1BN_2$

$= 2a.$

II. Experimental

1. In the experiment with a parallel beam of light and a mirror we showed that, if we turned the mirror through an angle of 45°, the reflected beam was turned through an angle of 90° and was seen vertically above us on the ceiling.

2. We took a plane silvered mirror, protractor and pins. We drew a straight line AB on a sheet of paper and from a point O in AB drew a line OM making an angle of 30° with AB. We placed a pin at O and another one at P in a line which made an angle of 40° with OM. We placed the mirror with its edge along AB.

An image of P was produced at Q_1. We put another pin R in a line with O and Q_1.

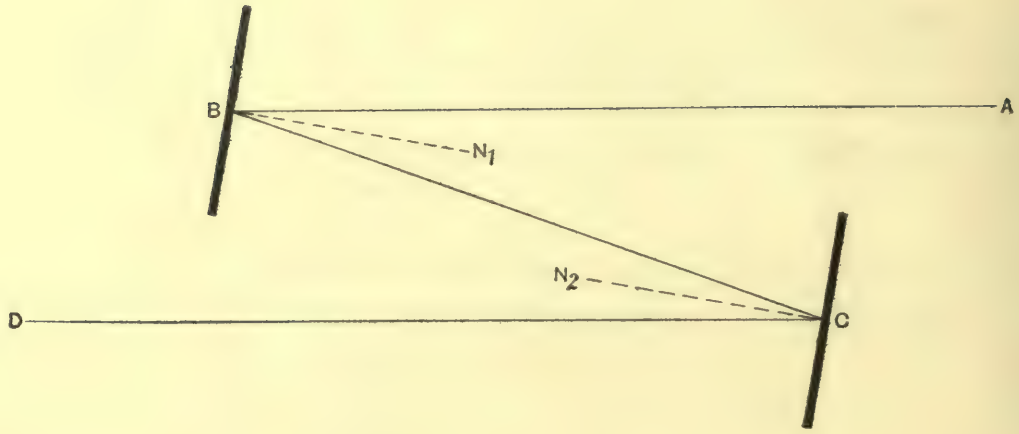
Then we turned the mirror until it was along the line OM and so obtained a fresh image Q_2. We put another pin S in a line with O and Q_2.

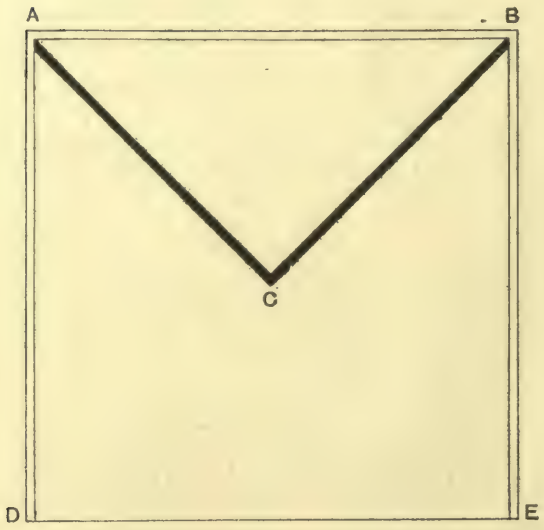
We measured the angle between OR and OS. This is clearly the angle through which the reflected ray has been turned.

Angle through which mirror has been turned = 30°.

Angle through which reflected ray has been turned = 60°.

This principle has many important applications, as, for instance, in "Mirror Galvanometers."


B
A
N_1
N_2
D
C


A
B
C
D
E

To find the action of two parallel mirrors upon a ray of light

We drew a ray so that it was reflected in turn from two parallel mirrors. The direction after double reflection was parallel to the original direction.

An arrangement of two parallel mirrors in this way provides one form of "periscope."

The Sextant

(See advanced notes.)

Optical Illusions

The Magic Cabinet is a good example of how optical illusions may be produced.

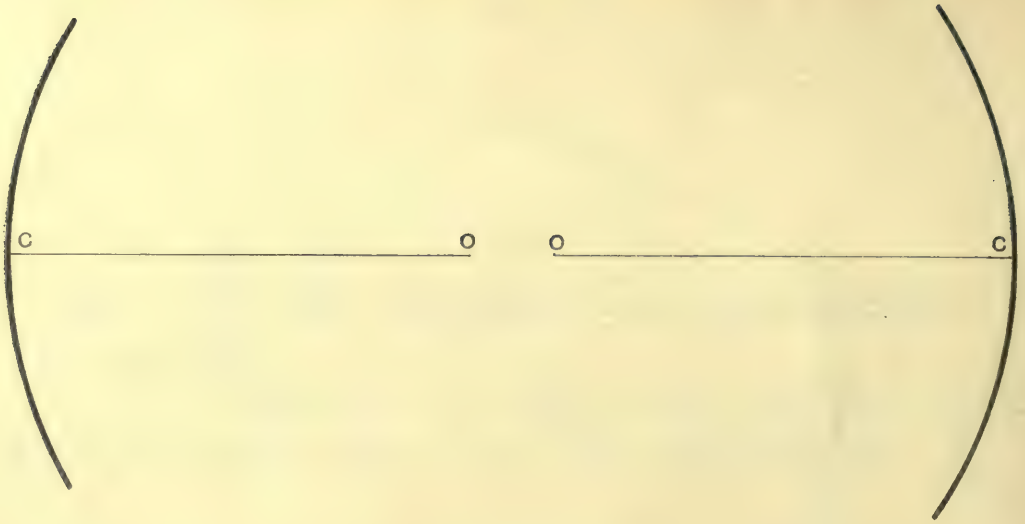
Two vertical silvered mirrors are hinged at the back corners of a cabinet. They meet at right angles and make angles of 45° at the back of the cabinet.

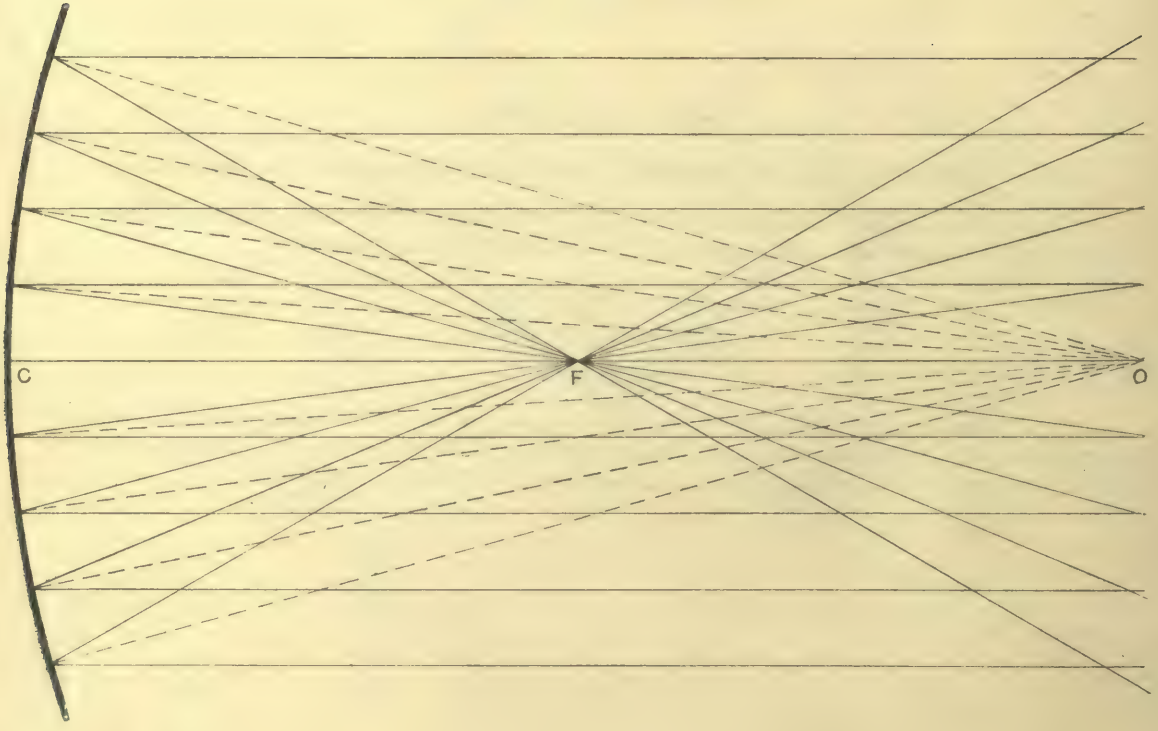
Let AB = back of cabinet.

AD, BE = sides of cabinet.

AC, CB = mirrors.

The sides of the cabinet by reflection appear to form the back and give the impression of a complete cabinet. Thus a performer can disappear by getting behind the mirrors and, on raising his head, it appears to be suspended in mid-air.


C
O
O
C


C
F
O

SPHERICAL MIRRORS

Spherical mirrors are parts of spheres. Every part is at the same distance from a fixed point, the centre of curvature, which corresponds to the centre of the sphere of which the mirror forms a part.

There are two kinds of mirrors:—

1. Concave, where the bright side of the mirror is *towards* the centre of curvature.
2. Convex, where the bright side of the mirror is *away from* the centre of curvature.

Def.

"The centre of curvature of a mirror is the centre of the sphere of which the mirror forms a part."

"The centre of face of a mirror is the middle point of the surface of the mirror."

"The principal axis of a mirror is the line joining the centre of curvature to the centre of face."

It is usual to denote the position of the centre of curvature by the letter *O* and the centre of face by the letter *C*.

To draw a normal to a spherical mirror

Since the radius of a sphere is perpendicular to the surface at any point, the normal at any point of a mirror is the radius at that point or the line joining the point to the centre of curvature.

Experiment to show that a point near the principal axis of a concave mirror has an image

We arranged the arc lamp opposite a small concave mirror, at a distance of about 120 cms., and obtained a sharp real image of the arc on a piece of paper about 40 cms. from the mirror. The light, after reflection at the mirror, forms a circular pencil, which converges to a point, forming the image, and then diverges again. On turning the mirror slightly, so that the arc was *not* near the principal axis, we obtained an elliptical pencil and no sharp image, the rays not converging to a point. In this case we get what are called "focal lines" and "a circle of least confusion."

To find the principal focus of a concave mirror by drawing

We drew a concave mirror of radius 15 cms. and a number of rays, parallel to the principal axis and 1 cm. apart. We drew the reflected rays, making the angle of incidence equal to the angle of reflection. We found that the reflected rays passed through a point on the principal axis, which is half-way between the centre of curvature and the centre of face. This is only true for the rays near the principal axis. The point to which the rays converge is called the "principal focus" of the mirror.

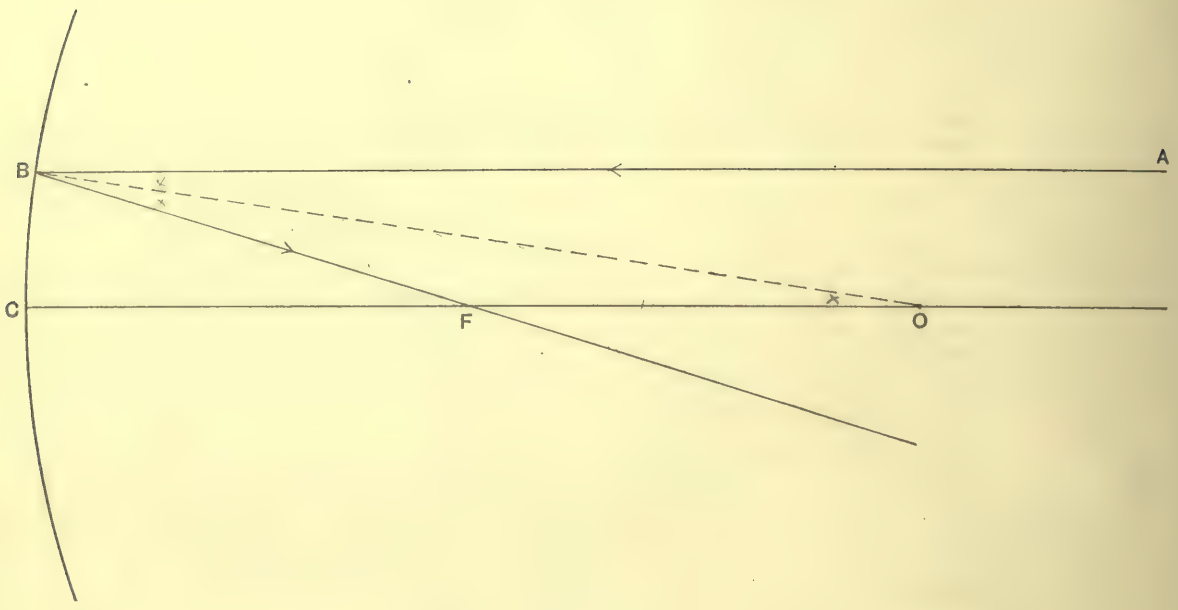
Def.

"The principal focus of a concave mirror is a point on the principal axis *to* which rays, parallel to the principal axis, *converge* after reflection."

It is usual to denote the position of the principal focus by the letter *F*.

Def.

"The focal length of a mirror is the distance of the principal focus from the centre of face."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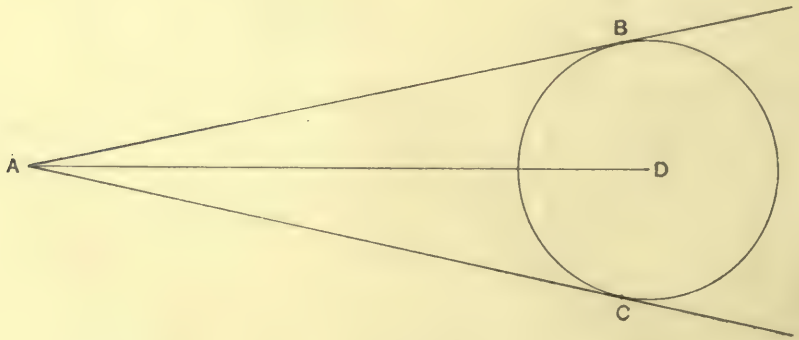
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Proof of the position of the Principal Focus

1. Theoretical.

In the diagram,

$$\angle ABO = \angle FBO.$$
$$\angle ABO = \angle BOF.$$
$$\therefore \angle FBO = \angle BOF.$$
$$\therefore FB = FO.$$

But, when B is near to C,

$$FB = FC \text{ nearly.}$$
$$\therefore FC = FO.$$

i.e. F lies half-way between O and C.

2. Experimental.

We took a concave mirror and arranged in front of it a knitting needle on a cork in such a position that it coincided with its own image. This point is the centre of curvature of the mirror. We measured the distance of the point from the mirror.

Distance = 60·5 cms.

∴ Radius of curvature = 60·5 cms.

Then we placed the needle a long distance away, so that the rays from it were nearly parallel and came to a focus at the principal focus. We found the position of the image by putting a second needle to coincide with it. This point is the principal focus of the mirror. We measured the distance of the point from the mirror.

Distance = 31·3 cms.

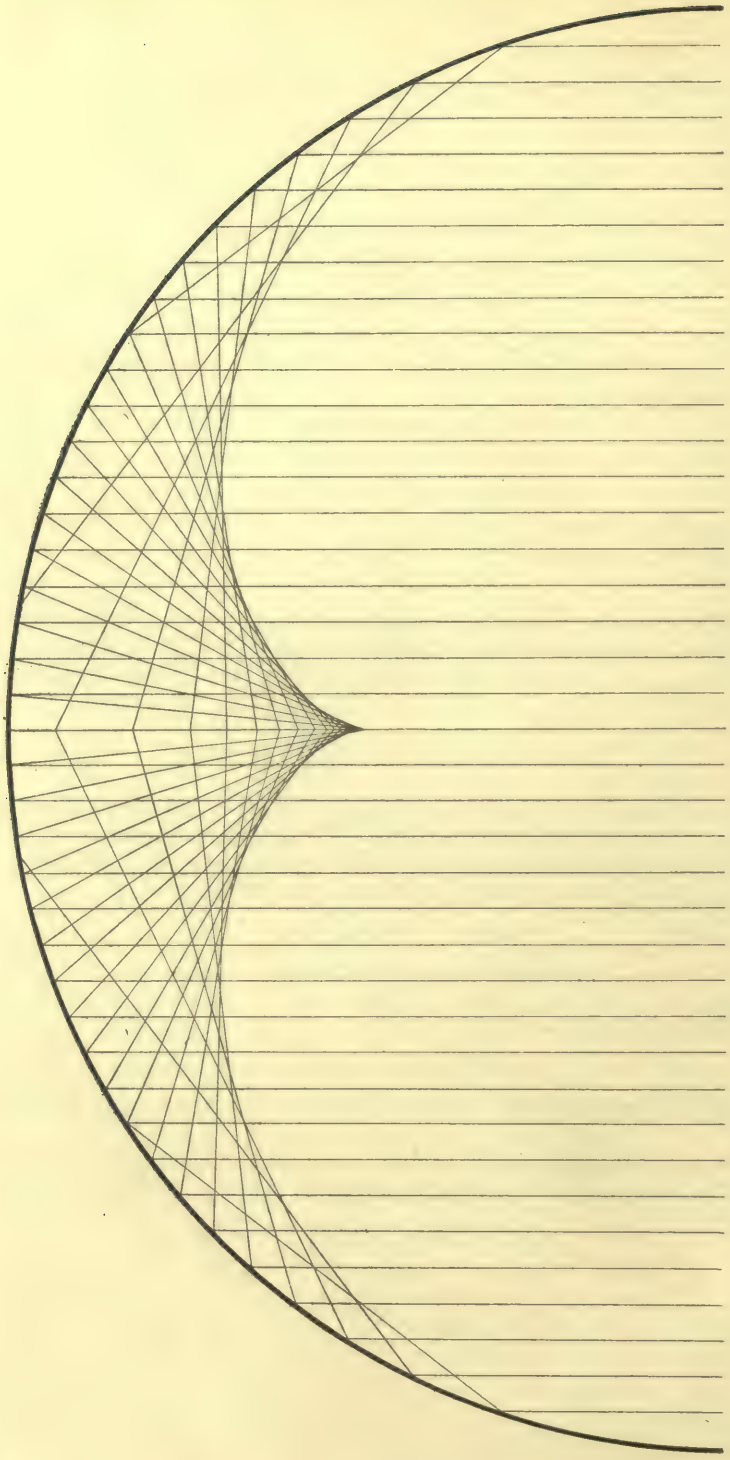
∴ Focal length = 31·3 cms.

We compared the two results.

Construction for drawing the reflected ray in the case of a mirror

If two tangents AB, AC are drawn to a circle from an external point A, they make equal angles with the straight line which joins the point A to the centre of the circle, D.

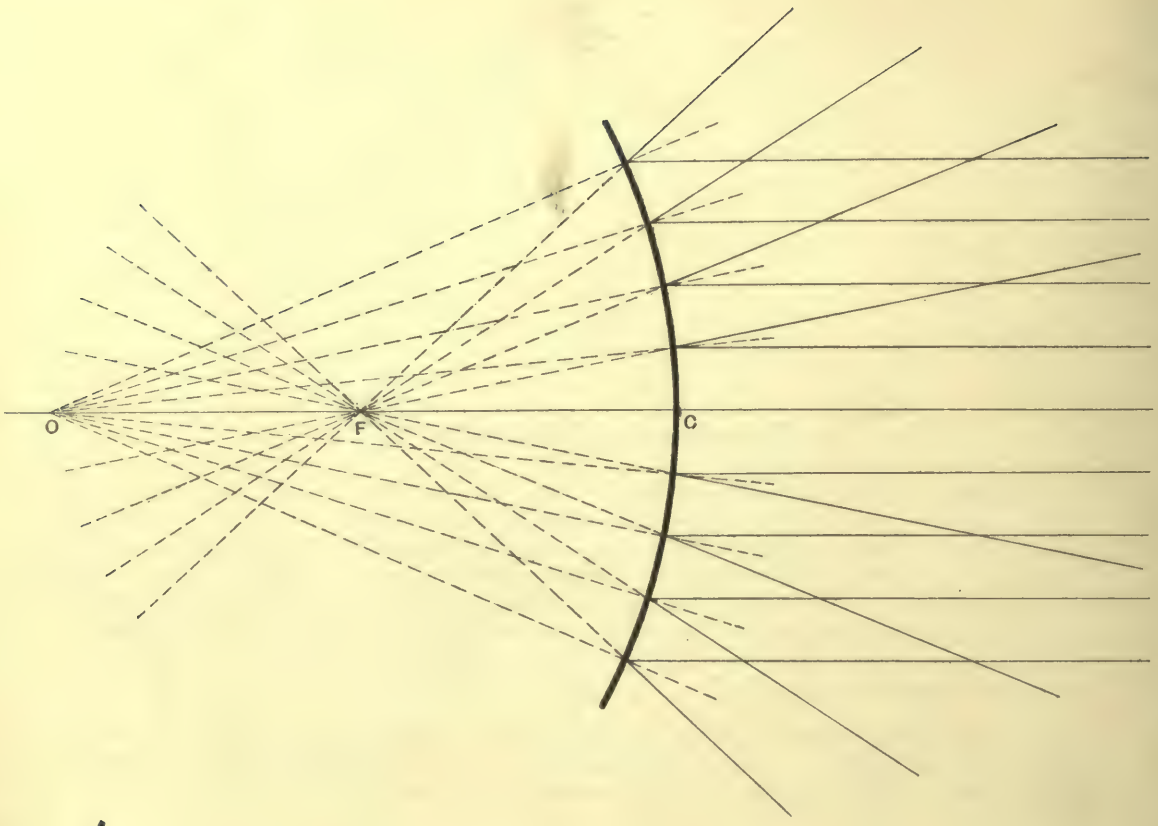
i.e. $\angle BAD = \angle CAD.$



To draw the Caustic Curve

We drew a mirror of radius 10 cms., making it a complete semicircle, and a number of rays parallel to the principal axis. Making use of the "circle method" we drew the reflected rays. We found that they did not all pass through the principal focus, only those near the principal axis doing so. The reflected rays are tangents to a curve called the "caustic curve," which can be drawn by joining up the points *where two successive reflected rays meet.* It is called the caustic curve because, since the rays are concentrated along it, burning will take place there. It is frequently to be seen in curved vessels. *e.g.* glasses, cups, basins, etc.

It follows from this that the principal focus is only a fixed position in a *small* mirror, *i.e.* one which is a small fraction of a sphere. In finding the position of images formed by mirrors we therefore assume *that the mirror is only a small portion of a sphere.*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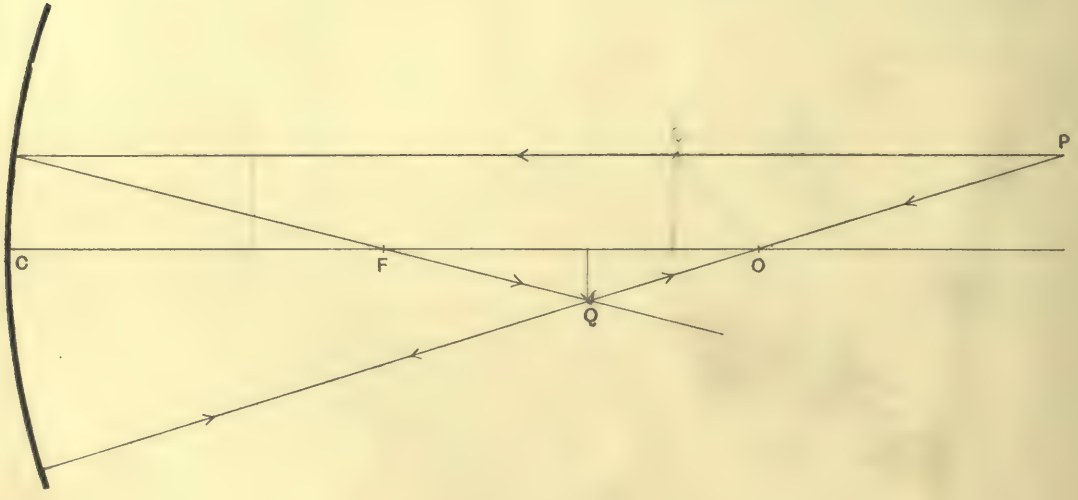
P
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To find the principal focus of a convex mirror by drawing

We drew a convex mirror of radius 10 cms. and a number of rays, parallel to the principal axis and 1 cm. apart. We drew the reflected rays, making the angle of incidence equal to the angle of reflection. We found that the reflected rays *appeared to diverge* from a point on the principal axis, which is half-way between the centre of curvature and the centre of face. This point is called the "principal focus" of the mirror. We noted that it was *behind* the mirror.

Def.

"The principal focus of a convex mirror is a point on the principal axis *from* which rays, parallel to the principal axis, *appear to diverge* after reflection."

Convention as to signs

Lines drawn to the *right* of a mirror are taken as *positive*, whilst those drawn to the *left* are taken as *negative*.

To find, by drawing, the image of a point formed by reflection in a concave mirror

We have seen in a previous experiment that any point near the principal axis of a concave mirror has an image, *i.e.* the rays diverging from the point, after reflection, converge to a second point, which is called the real image of the first point.

To find this point it will be sufficient to trace the course of *two rays* and find where they intersect one another after reflection.

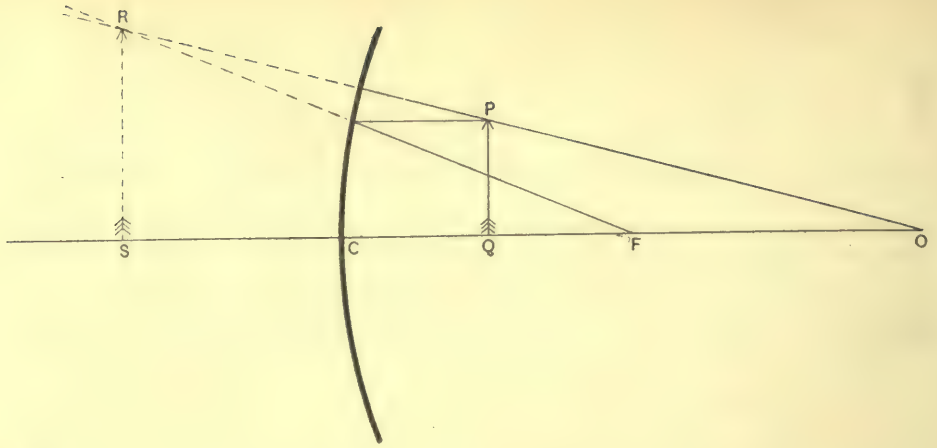
The two rays chosen are:—

1. A ray parallel to the principal axis, which, after reflection, passes through the principal focus.

2. A ray through the centre of curvature, which, after reflection, passes back along the same line.

Thus, in the diagram, Q is the real image of P.

Note that, if we drew any other ray from P to the mirror, it would also pass through Q after reflection.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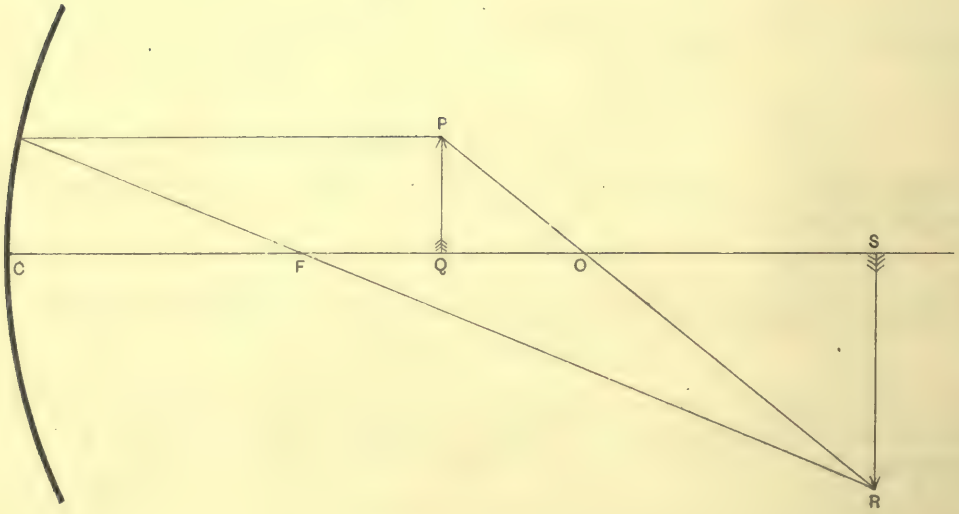
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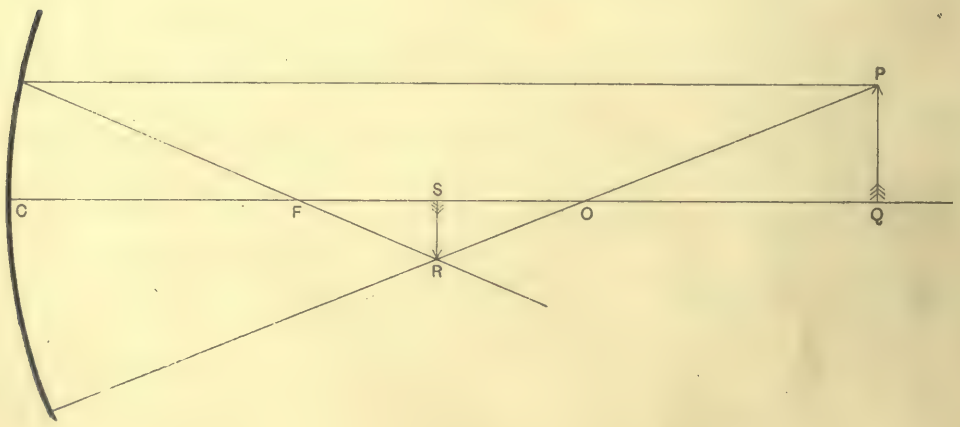
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To find by drawing the image of an object formed by reflection in a concave mirror

There are three cases to consider :—

1. When the object is between the centre of face and the principal focus.

   The image is

   Virtual.
   Erect.
   Magnified.
   Behind mirror.

2. When the object is between the principal focus and the centre of curvature.

   The image is

   Real.
   Inverted.
   Magnified.
   Outside *O*.

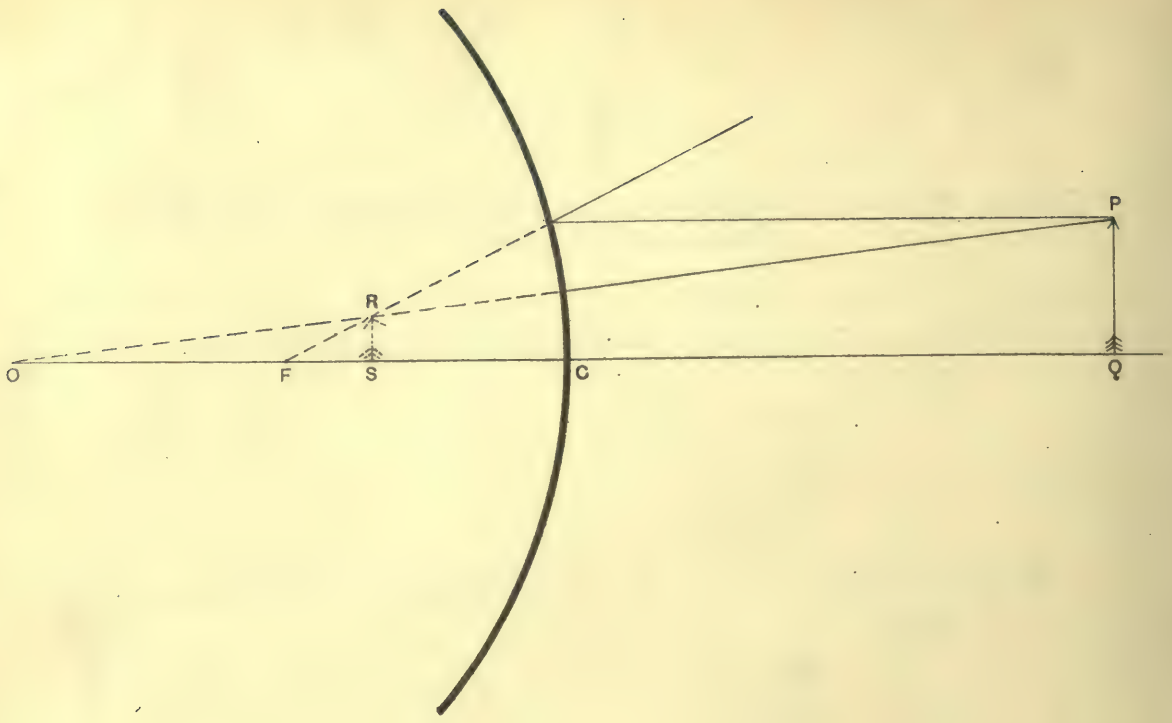
3. When the object is outside the centre of curvature.

   The image is

   Real.
   Inverted.
   Diminished.
   Between *F* and *O*.

Confirmation by Experiment

We took a candle, put it in front of a concave mirror in the three positions mentioned and tested the three cases.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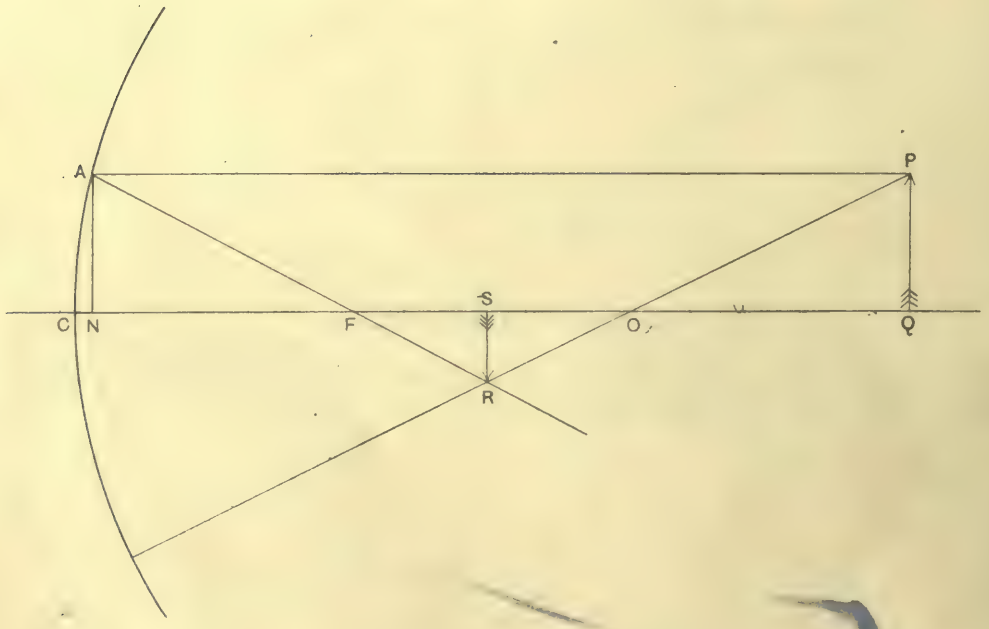
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To trace the nature of the image with a concave mirror as we pass from the mirror outwards

Object.	Image.
1. Between F and C.	Virtual, erect, magnified, behind mirror.
2. At F.	At infinity.
3. Between F and O.	Real, inverted, magnified, outside O.
4. At O.	Coincides with object.
5. Outside O.	Real, inverted, diminished, between F and O.

To find by drawing the image of an object formed by reflection in a convex mirror

The construction is almost the same as for a concave mirror, the only difference being that the centre of curvature and principal focus are behind the mirror, and the ray parallel to the principal axis appears to come from the principal focus.

The image is

Virtual.
Erect.
Diminished.
Behind mirror.

This is the *only* case.

Confirmation by Experiment

We took an object and put it in front of a convex mirror in various positions.

To obtain a formula for a concave mirror

We drew the case for a real, diminished image with a concave mirror.

Let u = distance of object from mirror.
v = distance of image from mirror.
r = radius of curvature of mirror.
f = focal length of mirror.

In the similar triangles POQ, ROS,

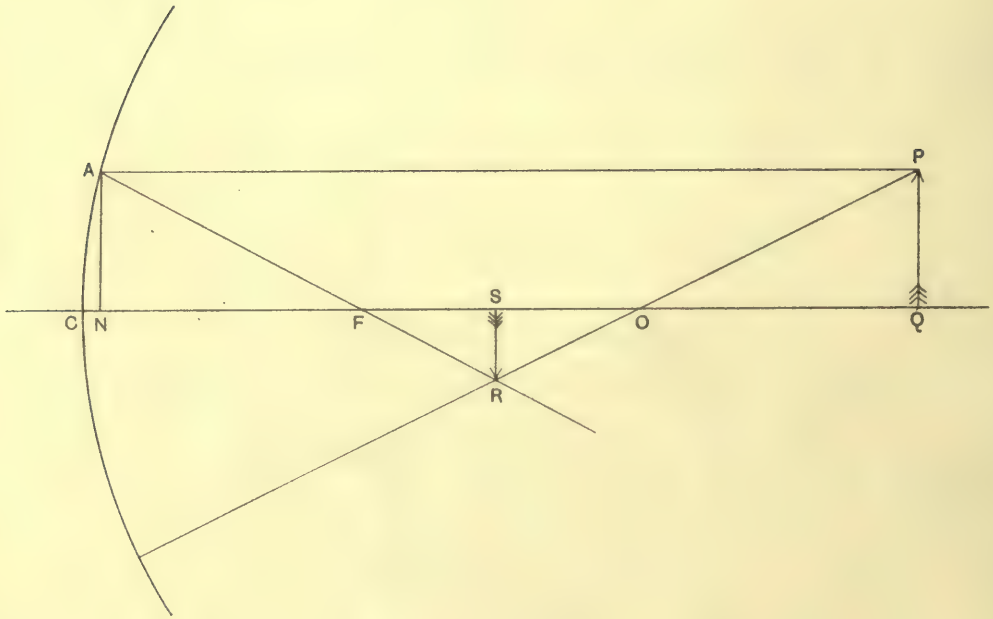
$$\frac{PQ}{RS} = \frac{OQ}{OS}. \qquad (1)$$

In the similar triangles AFN, RFS,

$$\frac{AN}{RS} = \frac{FN}{FS}.$$

But $AN = PQ$.

$$\therefore \frac{PQ}{RS} = \frac{FN}{FS}. \qquad (2)$$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From (1) and (2),

$$\frac{OQ}{OS} = \frac{FN}{FS}.$$

But when A is near to C, $FN = FC$.

$$\therefore \quad \frac{OQ}{OS} = \frac{FC}{FS},$$

$$\frac{CQ - CO}{CO - CS} = \frac{FC}{CS - CF},$$

$$\frac{u - r}{r - v} = \frac{\frac{r}{2}}{v - \frac{r}{2}},$$

$$\frac{u - r}{r - v} = \frac{\frac{r}{2}}{\frac{2v - r}{2}},$$

$$\frac{u - r}{r - v} = \frac{r}{2v - r},$$

$$(u - r)(2v - r) = r(r - v),$$

$$2uv + r^2 - ur - 2vr = r^2 - vr,$$

$$-vr - ur = -2uv,$$

$$vr + ur = 2uv.$$

Dividing through by uvr,

$$\frac{1}{u} + \frac{1}{v} = \frac{2}{r}.$$

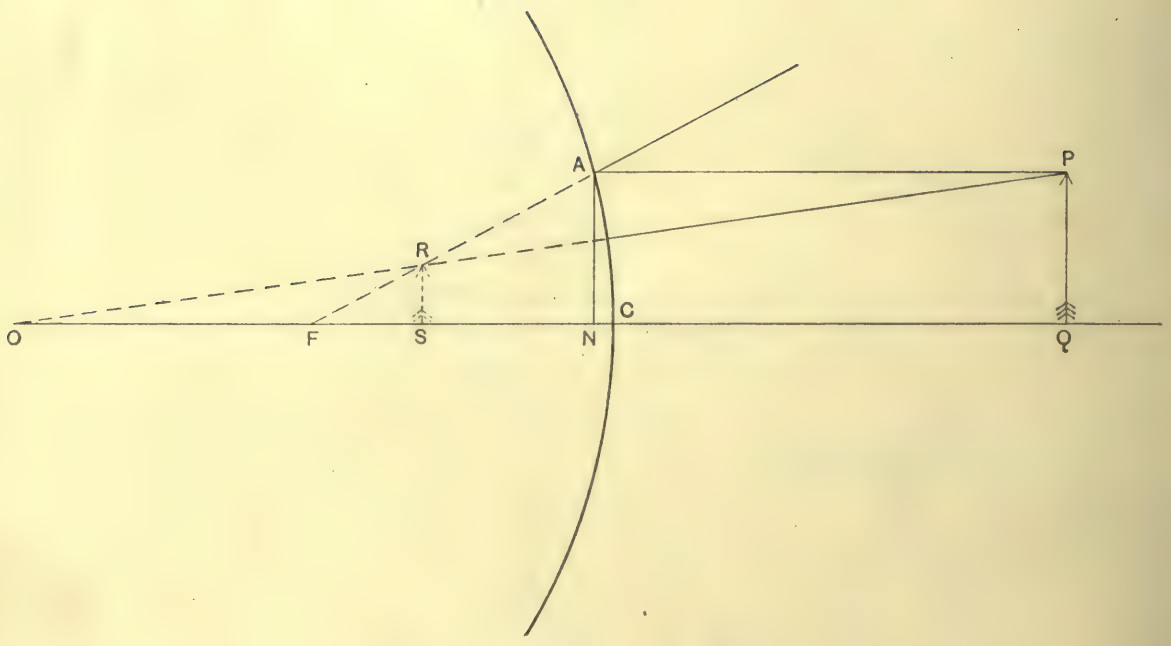
Since $f = \frac{r}{2}$;

$$\frac{1}{f} = \frac{1}{\frac{r}{2}}$$

$$= \frac{2}{r}.$$

$$\therefore \quad \frac{1}{u} + \frac{1}{v} = \frac{1}{f}.$$

This is the *standard* formula for mirrors and is applicable in *all* cases of mirrors, if we remember to change the signs.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Rule for employing the formula $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$

This formula can be used in *all* cases of mirrors, if we remember that, when v and f are behind the mirror, they must be given the *negative* sign.

Concave mirror,

1. $\frac{1}{u}-\frac{1}{v}=\frac{1}{f}$.

2. $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$.

3. $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$.

Convex mirror,

$$\frac{1}{u}-\frac{1}{v}=-\frac{1}{f}.$$

To obtain a formula for a convex mirror

We drew the case for a convex mirror.

In the similar triangles POQ, ROS,

$$\frac{PQ}{RS}=\frac{OQ}{OS}. \qquad (1)$$

In the similar triangles AFN, RFS,

$$\frac{AN}{RS}=\frac{FN}{FS}.$$

But $AN=PQ$.

$$\therefore \frac{PQ}{RS}=\frac{FN}{FS}. \qquad (2)$$

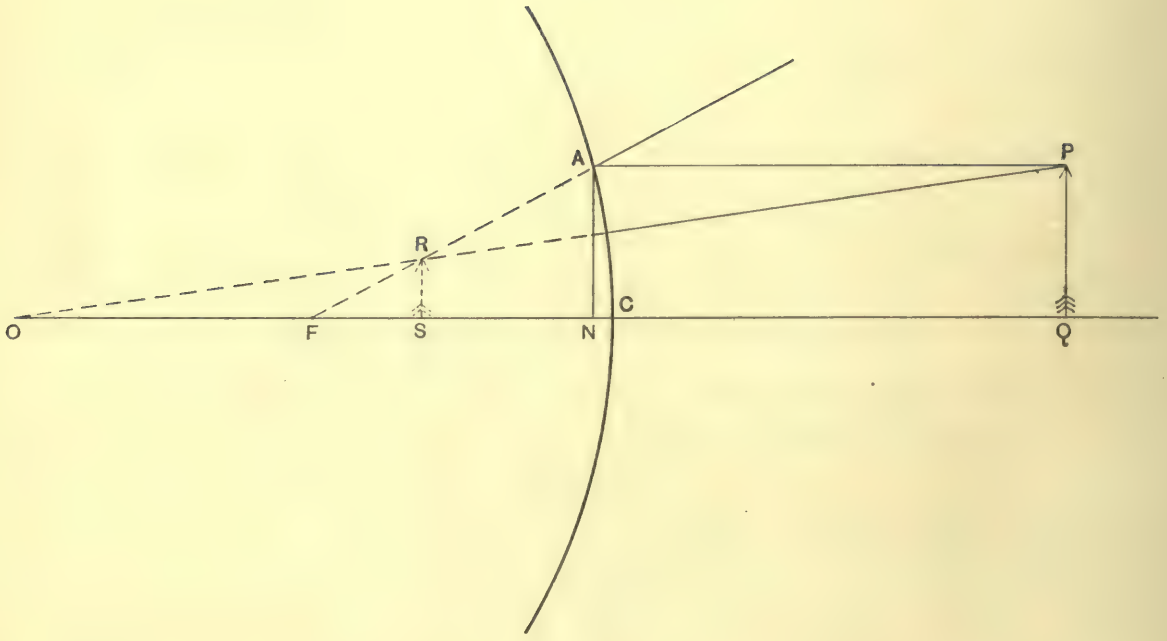
From (1) and (2),

$$\frac{OQ}{OS}=\frac{FN}{FS}.$$

But when A is near to C, $FN=FC$.

$$\therefore \frac{OQ}{OS}=\frac{FC}{FS},$$

$$\frac{OC+CQ}{OC-SC}=\frac{FC}{FC-SC}.$$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$$\frac{r+u}{r-v}=\frac{\frac{r}{2}}{\frac{r}{2}-v},$$

$$\frac{r+u}{r-v}=\frac{\frac{r}{2}}{\frac{r-2v}{2}},$$

$$\frac{r+u}{r-v}=\frac{r}{r-2v},$$

$$(r+u)(r-2v)=r(r-v),$$

$$r^2-2uv+ur-2vr=r^2-vr,$$

$$-vr+ur=2uv,$$

$$vr-ur=-2uv.$$

Dividing through by uvr,

$$\frac{1}{u}-\frac{1}{v}=-\frac{2}{r}.$$

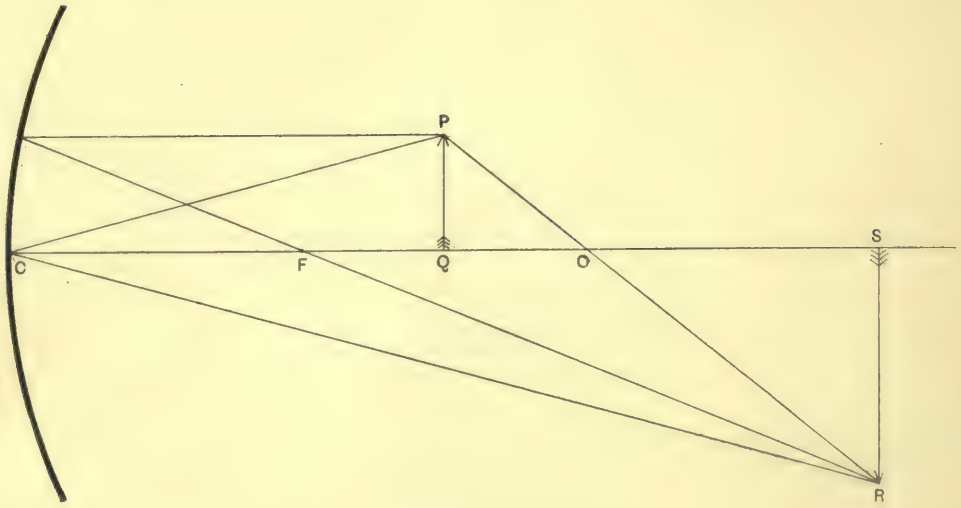
$$\therefore\ \frac{1}{u}-\frac{1}{v}=-\frac{1}{f}.$$

Note that this formula can be obtained from the *standard* formula

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$$

if we change the signs of v and f, which are behind the mirror, when we get

$$\frac{1}{u}-\frac{1}{v}=-\frac{1}{f}.$$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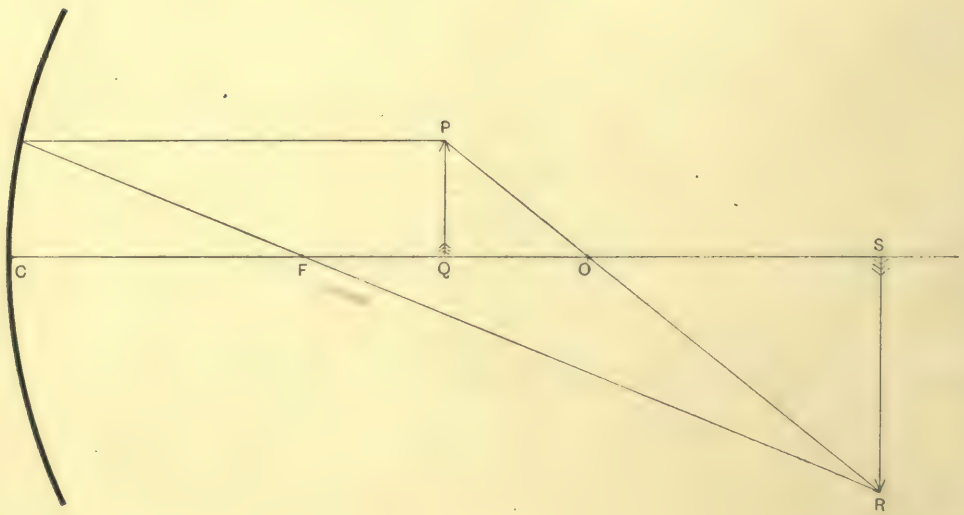
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To find the magnification produced by a mirror

We drew the case for a real, magnified image with a concave mirror.

We joined PC, RC.

Then in the similar triangles RCS, PCQ,

$$\frac{RS}{PQ} = \frac{SC}{QC} = \frac{v}{u}.$$

$$\text{Magnification} = \frac{\text{size of image}}{\text{size of object}}$$

$$= \frac{RS}{PQ}$$

$$= \frac{v}{u}.$$

To find the position for the object with a concave mirror so that the real image obtained shall be twice the size of the object

Since the image is twice the size of the object,

$$\frac{v}{u} = 2.$$

$$\therefore\ v = 2u.$$

$$\text{Then } \frac{1}{u} + \frac{1}{v} = \frac{1}{f}.$$

$$\therefore\ \frac{1}{u} + \frac{1}{2u} = \frac{1}{f},$$

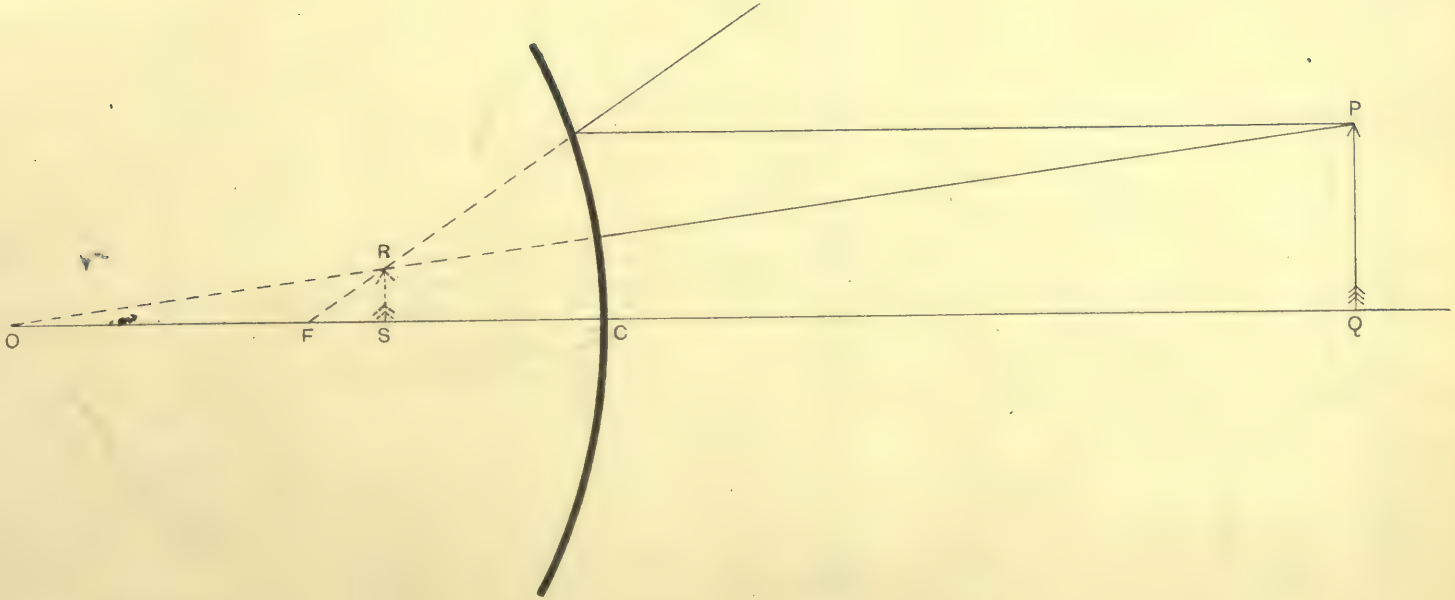
$$\frac{2+1}{2u} = \frac{1}{f},$$

$$\frac{3}{2u} = \frac{1}{f},$$

$$2u = 3f.$$

$$\therefore\ u = \frac{3}{2}f,$$

i.e. the object must be placed half-way between F and O.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Standard Type of Example

Find by drawing and calculation the position and size of the image of an object formed by reflection in a convex mirror, given the following data:—

Radius of curvature = 8 cms.

Distance of object from mirror = 10 cms.

Size of object = 2·5 cms.

The drawing must be done across the page.

1. By measurement.

   Distance of image from mirror = 2·95 cms.

   Size of image = ·70 cm.

2. By calculation.

$$\frac{1}{u}-\frac{1}{v}=-\frac{1}{f}.$$

$$\therefore -\frac{1}{v}=-\frac{1}{f}-\frac{1}{u}.$$

$$\therefore \frac{1}{v}=\frac{1}{f}+\frac{1}{u}$$

$$=\frac{1}{4}+\frac{1}{10}$$

$$=\frac{5+2}{20}$$

$$=\frac{7}{20}.$$

$$\therefore v=\frac{20}{7}$$

$$=2{\cdot}86 \text{ cms.}$$

Distance of image from mirror = 2·86 cms.

$$\frac{\text{Size of image}}{\text{Size of object}}=\frac{v}{u}.$$

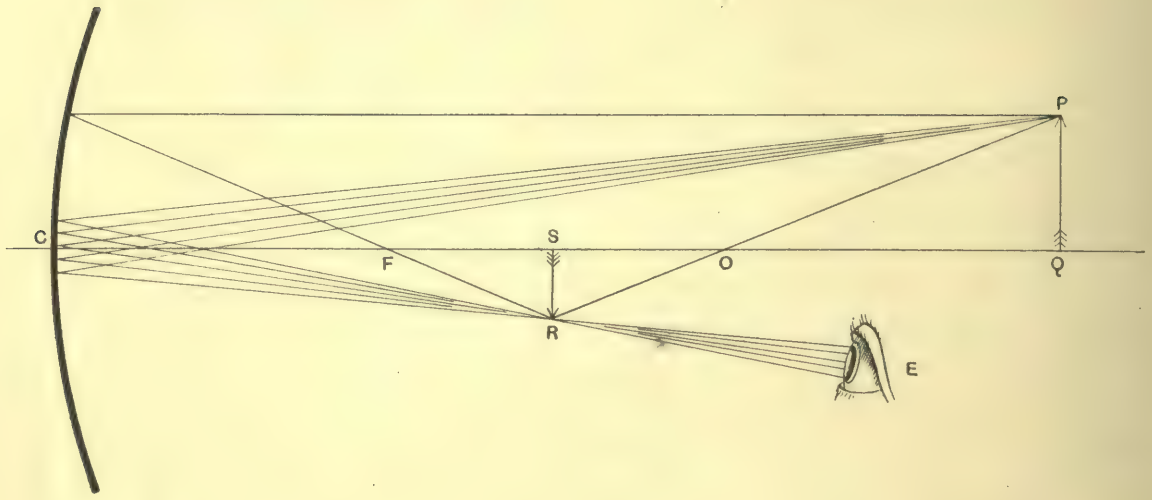
$$\therefore \text{Size of image} = \text{size of object} \times \frac{v}{u}$$

$$=2{\cdot}5\times\frac{2{\cdot}86}{10}$$

$$=\frac{7{\cdot}14}{10}$$

$$=\cdot 714 \text{ cm.}$$

Size of image = ·714 cm.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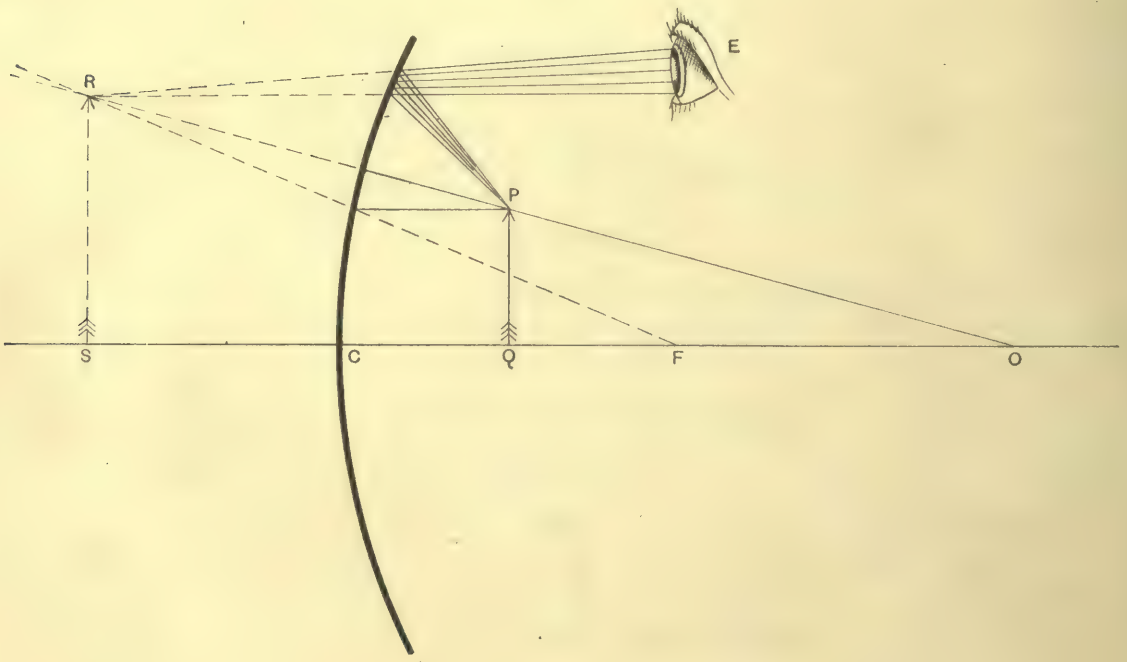
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To draw the rays by which an eye sees the image of a point formed by reflection in a concave mirror

I. Real Image

We drew a cone of rays, with the eye as base, to the image and produced the rays to the mirror. We joined up the points, where the rays meet the mirror, to the object.

II. Virtual Image

We drew a cone of rays, with the eye as base, to the image, dotting them where they do not exist. We joined up the points, where the rays meet the mirror, to the object.

To find the focal length of a concave mirror

There are three methods:—

1. We placed a knitting needle in front of a concave mirror, so that the object and image coincided in all positions of the eye. This needle was at the centre of curvature. We measured the distance of it from the mirror.

$$\text{Distance} = 60{\cdot}2 \text{ cms.}$$

$$\therefore \text{ Radius of curvature} = 60{\cdot}2 \text{ cms.}$$

$$\therefore \text{ Focal length} = \frac{60{\cdot}2}{2}$$

$$= 30{\cdot}1 \text{ cms.}$$

$$\text{Focal length of mirror} = 30{\cdot}1 \text{ cms.}$$

2. We placed a knitting needle, P, between the principal focus and centre of curvature, and arranged a second needle, Q, outside the centre of curvature to coincide with the image of P seen in the mirror.

We measured u and v.

$$u = 52{\cdot}3 \text{ cms.}$$

$$v = 72{\cdot}4 \text{ cms.}$$

$$\text{Then } \frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$= \frac{1}{52{\cdot}3} + \frac{1}{72{\cdot}4}$$

$$= \frac{72{\cdot}4 + 52{\cdot}3}{52{\cdot}3 \times 72{\cdot}4}$$

$$= \frac{124{\cdot}7}{3786{\cdot}52}$$

$$\therefore \quad f = \frac{3786{\cdot}52}{124{\cdot}7}$$

$$= 30{\cdot}4 \text{ cms.}$$

$$\text{Focal length of mirror} = 30{\cdot}4 \text{ cms.}$$

3. We placed a knitting needle, P, between the centre of face and principal focus, and arranged a second needle, Q, behind the mirror to coincide with the image of P seen in the mirror. This we were able to do by using a mirror with a strip of silvering off the back.

We measured u and v.

$$u = 18{\cdot}3 \text{ cms.}$$
$$v = 48{\cdot}0 \text{ cms.}$$

Then $$\frac{1}{f} = \frac{1}{u} - \frac{1}{v}$$
$$= \frac{1}{18{\cdot}3} - \frac{1}{48{\cdot}0}$$
$$= \frac{48{\cdot}0 - 18{\cdot}3}{18{\cdot}3 \times 48{\cdot}0}$$
$$= \frac{29{\cdot}7}{878{\cdot}4}.$$
$$\therefore f = \frac{878{\cdot}4}{29{\cdot}7}$$
$$= 29{\cdot}6 \text{ cms.}$$

Focal length of mirror = 29·6 cms.

To find the focal length of a convex mirror

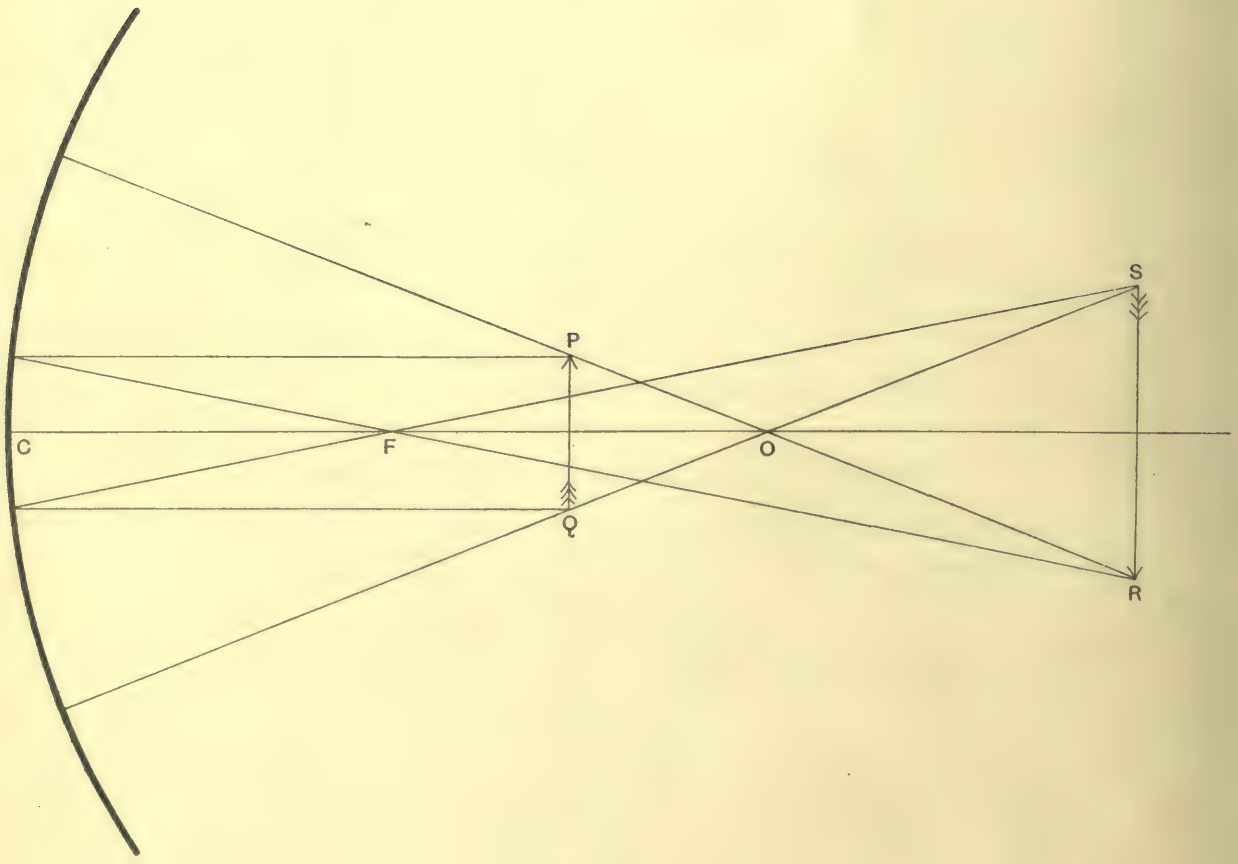
We employed the third method for a concave mirror.

We measured u and v.

$$u = 26{\cdot}8 \text{ cms.}$$
$$v = 14{\cdot}2 \text{ cms.}$$

Then $$\frac{1}{u} - \frac{1}{v} = -\frac{1}{f}.$$
$$\therefore \frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$
$$= \frac{1}{14{\cdot}2} - \frac{1}{26{\cdot}8}$$
$$= \frac{26{\cdot}8 - 14{\cdot}2}{14{\cdot}2 \times 26{\cdot}8}$$
$$= \frac{12{\cdot}6}{380{\cdot}56}.$$
$$\therefore f = \frac{380{\cdot}56}{12{\cdot}6}$$
$$= 30{\cdot}2 \text{ cms.}$$

Focal length of mirror = 30·2 cms.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Practical points to be noticed in finding the position of Images

1. To test whether an image is behind or in front of a mirror, move the eye across the mirror. If the image moves with the eye across the mirror, it is behind; if it goes away from the eye, it is in front.

2. To find the position of an image in front of a mirror, obtain two positions such that in one, on moving the eye to the *left*, the image moves to the *right more rapidly* than the second needle and in the other *less rapidly*. Then the position of the image is somewhere between these two positions,

3. To find the position of an image behind a mirror, the same thing is done, the only difference being that the image and second needle move to the left, if the eye moves to the left.

To prove by experiment that magnification $= \frac{v}{u}$

We took two needles, P and Q, and placed them between the principal focus and centre of curvature of a concave mirror, at the same distance from the mirror and about 4 cms. apart. We then placed two other needles, R and S, to coincide with the images of the first two, and measured the distance between them, which gave the size of the image. We also measured u and v.

We obtained the following results:—

$$\text{Size of object} = 4{\cdot}0 \text{ cms.}$$

$$\text{Size of image} = 7{\cdot}1 \text{ cms.}$$

$$u = 44{\cdot}3 \text{ cms.}$$

$$v = 78{\cdot}2 \text{ cms.}$$

$$\text{Magnification} = \frac{\text{size of image}}{\text{size of object}}$$

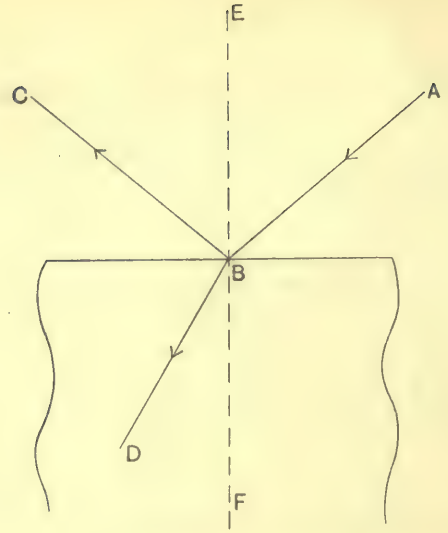
$$= \frac{7{\cdot}1 \text{ cms.}}{4{\cdot}0 \text{ cms.}}$$

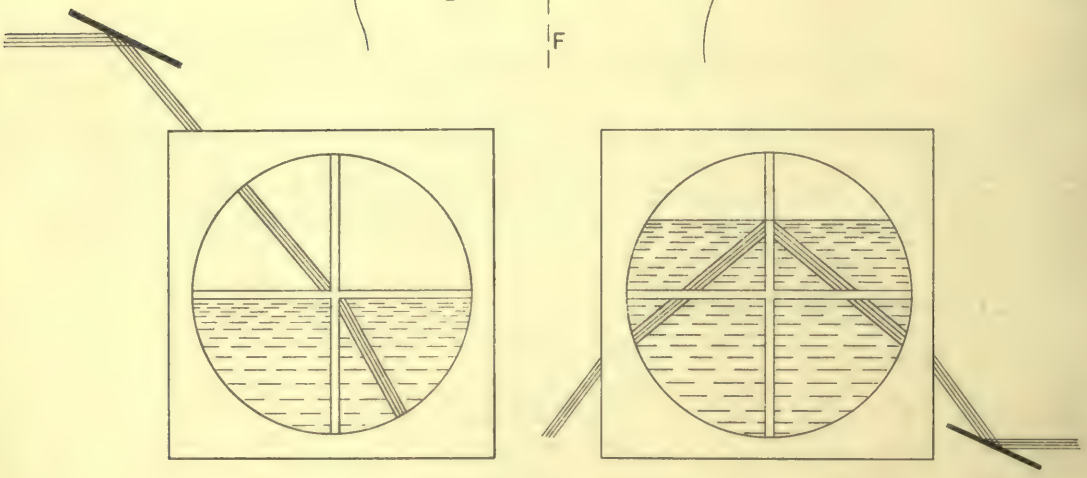
$$= 1{\cdot}78.$$

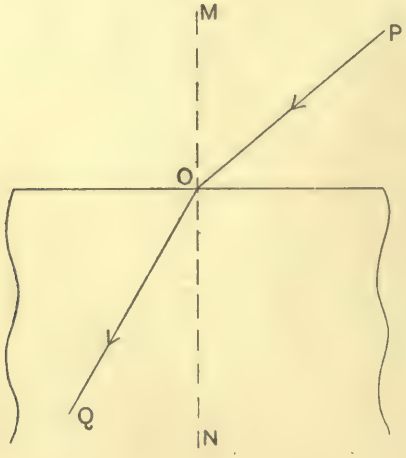
$$\frac{v}{u} = \frac{78{\cdot}2 \text{ cms.}}{44{\cdot}3 \text{ cms.}}$$

$$= 1{\cdot}77.$$

$$\therefore \text{ Magnification} = \frac{v}{u}.$$


E
C
A
B
D
F




M
P
O
Q
N

REFRACTION OF LIGHT

If a ray of light, travelling in any medium, comes in contact *obliquely* with another medium, part of it is reflected and part refracted.

In the diagram

AB = incident ray.
BC = reflected ray.
BD = refracted ray.
$\angle ABE$ = angle of incidence.
$\angle DBF$ = angle of refraction.

Experiments to show the Refraction of Light

1. A parallel beam of light from a lantern was reflected by a mirror on to the surface of some coloured water in a trough.

The beam of light as it entered the water was bent *towards the normal.*

2. We arranged the trough so that a beam of light entered it at the side. It did *not* emerge, but was "totally reflected."

3. A coin at the bottom of a trough full of water appeared to be raised. When the eye was placed on a level with the top of the water, the coin appeared to be almost on the surface.

4. A half-metre measure put in the water appeared to be bent.

Laws of Refraction

Def.

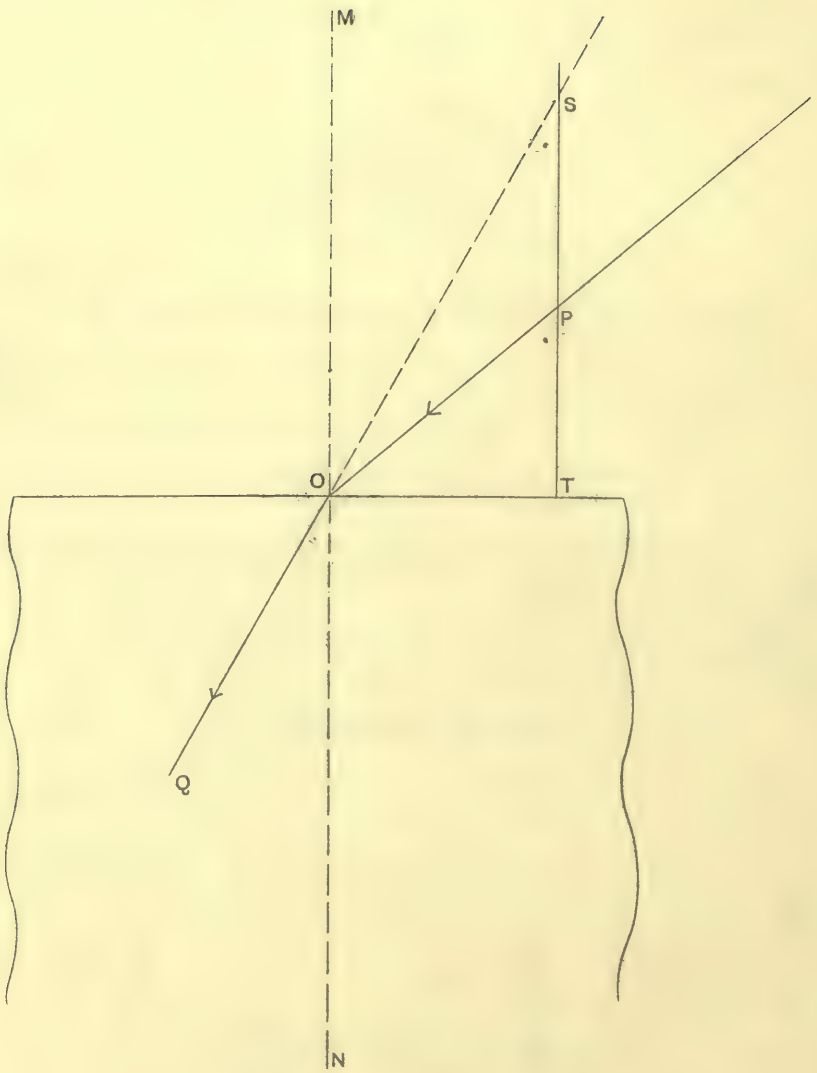
I. "The incident ray, the normal to the surface and the refracted ray lie in one plane."

II. "The sine of the angle of incidence bears to the sine of the angle of refraction a ratio, which is always the same for the same two media and is called the index of refraction."

Law II. says that, if PO is an incident ray and OQ the refracted ray, and if a normal MON is drawn at O, then

$$\frac{\text{sin angle of incidence}}{\text{sin angle of refraction}} = \frac{\sin POM}{\sin QON}$$
$$= \text{constant}$$
$$= \text{index of refraction}$$
$$= \mu.$$

This is true whatever the direction of the incident ray may be.


M
S
P
O
T
Q
N

Alternative Form of Law II

Let P be any point on an incident ray PO. Draw a normal PT to cut the refracted ray produced backwards at S.

$$\text{Then } \mu = \frac{\text{sin angle of incidence}}{\text{sin angle of refraction}}$$

$$= \frac{\sin POM}{\sin QON}$$

$$= \frac{\sin OPT}{\sin OST}$$

$$= \frac{\frac{OT}{OP}}{\frac{OT}{OS}}$$

$$= \frac{OS}{OP},$$

$$\mu = \frac{OS}{OP}.$$

$$\therefore OS = \mu OP.$$

Alternative Statement of Law II

Def.

"If any normal be drawn to cut the incident ray and the refracted ray produced backwards, the portions of these lines between the point of incidence and the normal bear to one another a ratio, which is always the same for the same two media and is called the index of refraction."

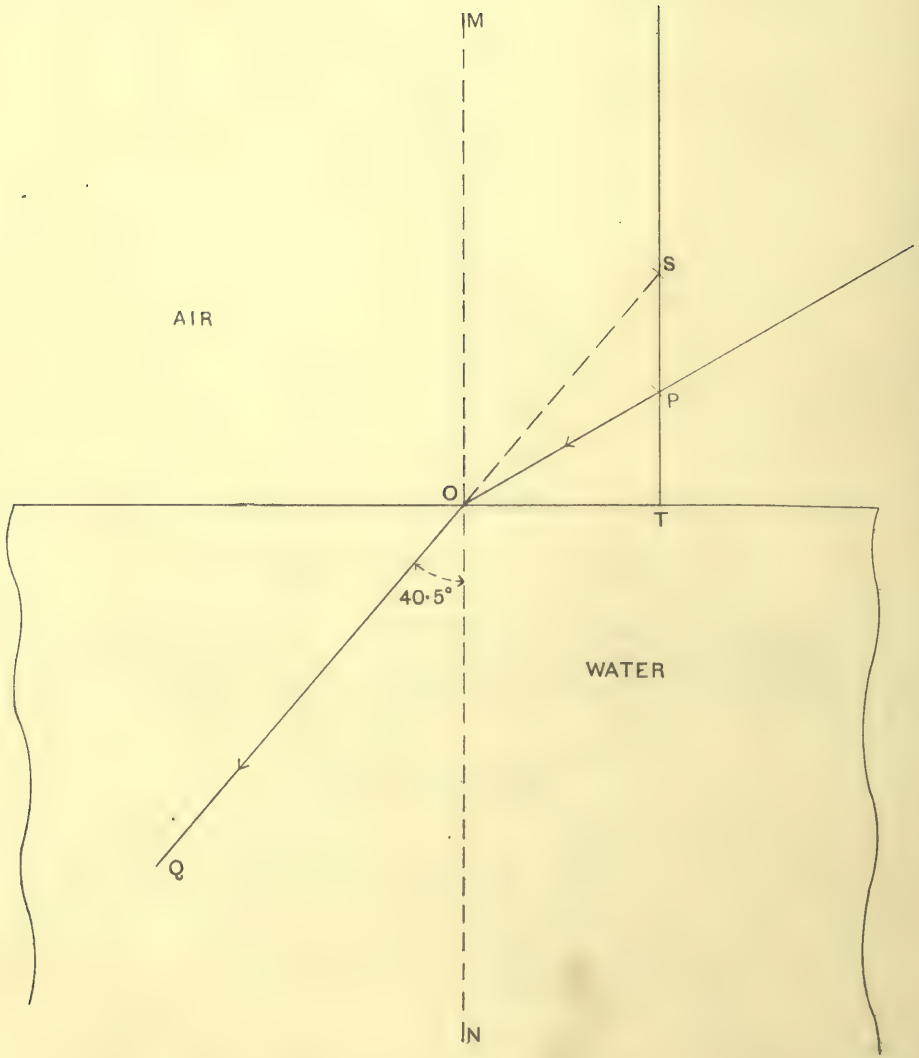
Refractive Indices

Diamond	2·42
Ruby	1·71
Carbon Bisulphide ...	1·63
Rock Salt	1·54
Crown Glass ...	1·50
Turpentine	1·46
Fluor Spar	1·43
Water	1·33

It is important to note that these are all for yellow light and from air to the medium.

The following are the most important refractive indices and *must be remembered.*

Air to glass, ${}_A\mu_G$ ...	$\frac{3}{2}$
Glass to air, ${}_G\mu_A$ ...	$\frac{2}{3}$
Air to water, ${}_A\mu_W$ ...	$\frac{4}{3}$
Water to air, ${}_W\mu_A$ ...	$\frac{3}{4}$


M
S
AIR
P
O
T
40·5°
WATER
Q
N

To draw a ray from air to water

Let angle of incidence = 60°.

We cut off from the incident ray a distance OP equal to 3 cms. and drew a normal to the surface through P. Then with centre O and radius

$$= \mu \times OP$$
$$= \tfrac{4}{3} \times OP$$
$$= \tfrac{4}{3} \times 3$$
$$= 4 \text{ cms.}$$

we drew part of a circle cutting the normal in S. We joined up SO with a dotted line and produced it onwards to Q.

Then OQ is the refracted ray. We measured the angle of refraction.

Angle of refraction = 40·5°.

To find the angle of refraction by calculation

$$\frac{\sin \text{ incidence}}{\sin \text{ refraction}} = \mu.$$

$$\therefore \sin \text{ refraction} = \frac{\sin \text{ incidence}}{\mu}$$

$$= \frac{\sin 60^\circ}{\mu}$$

$$= \frac{\cdot 8660}{\frac{4}{3}}$$

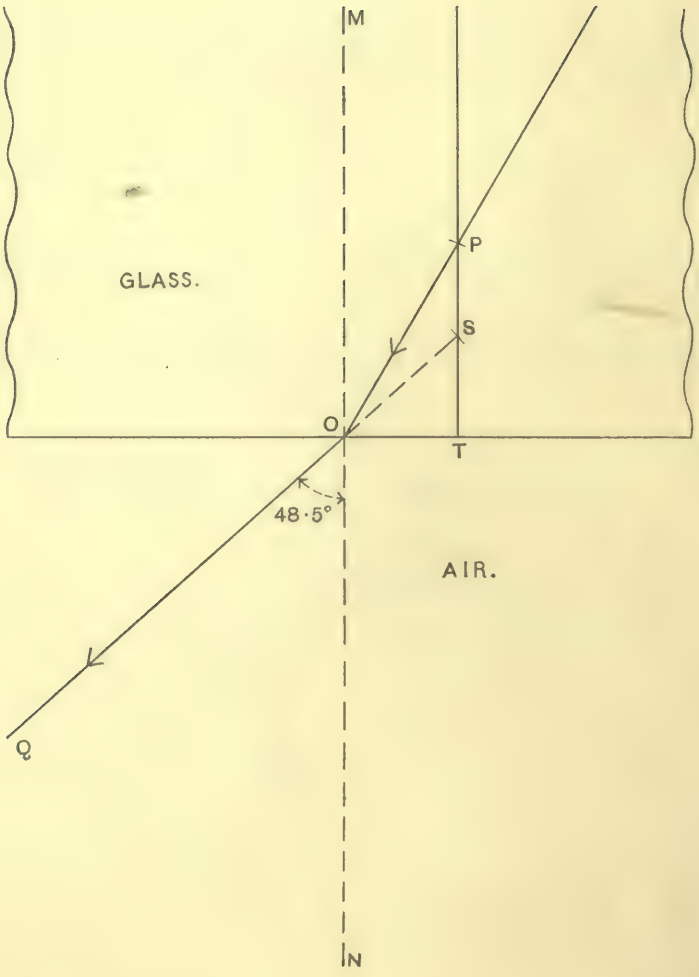
$$= \cdot 8660 \times \tfrac{3}{4}$$

$$= \frac{2 \cdot 5980}{4}$$

$$= \cdot 6495$$

$$= \sin 40 \cdot 5^\circ.$$

Angle of refraction = 40·5°.


M
GLASS.
P
S
O
T
48·5°
AIR.
Q
N

To draw a ray from glass to air

Let angle of incidence = 30°.

We cut off from the incident ray a distance OP equal to 3 cms. and drew a normal to the surface through P. Then with centre O and radius

$$= \mu \times OP$$
$$= \tfrac{2}{3} \times OP$$
$$= \tfrac{2}{3} \times 3$$
$$= 2 \text{ cms.}$$

we drew part of a circle cutting the normal in S. We joined up SO with a dotted line and produced it onwards to Q.

Then OQ is the refracted ray. We measured the angle of refraction.

Angle of refraction = 48·5°.

To find the angle of refraction by calculation

$$\frac{\text{sin incidence}}{\text{sin refraction}} = \mu.$$

$$\therefore \text{ sin refraction} = \frac{\text{sin incidence}}{\mu}$$

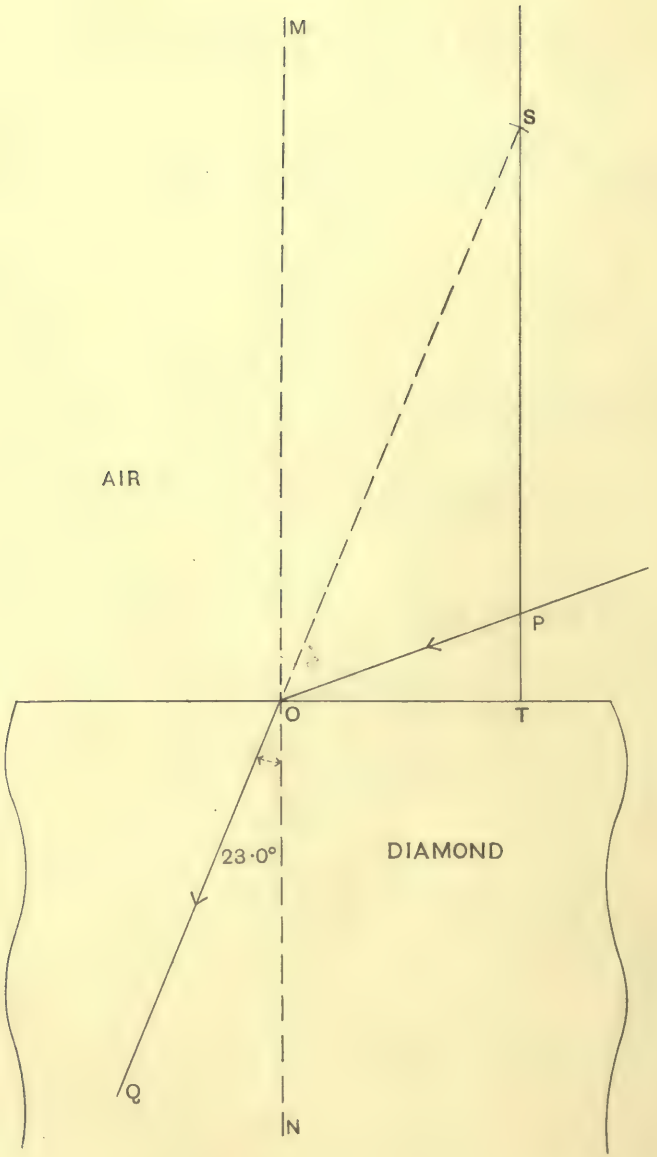
$$= \frac{\text{sin } 30°}{\mu}$$

$$= \frac{\cdot 5}{\frac{2}{3}}$$

$$= \cdot 75$$

$$= \text{sin } 48{\cdot}6°.$$

Angle of refraction = 48·6°.


M
S
AIR
P
O
T
23·0°
DIAMOND
Q
N

To draw a ray from air to diamond taking any normal

Let angle of incidence = 70°.

$${}_A\mu_D = 2{\cdot}42.$$

We drew *any* normal to cut the incident ray in P and measured OP.

$$OP = 3{\cdot}40 \text{ cms.}$$

Then with centre O and radius

$$= \mu \times OP$$
$$= 2{\cdot}42 \times 3{\cdot}40$$
$$= 8{\cdot}23 \text{ cms.}$$

we drew part of a circle cutting the normal in S. We joined up SO with a dotted line and produced it onwards to Q.

Then OQ is the refracted ray. We measured the angle of refraction.

Angle of refraction = 23·0°.

To find the angle of refraction by calculation

$$\frac{\sin \text{ incidence}}{\sin \text{ refraction}} = \mu.$$

$$\therefore \sin \text{ refraction} = \frac{\sin \text{ incidence}}{\mu}$$

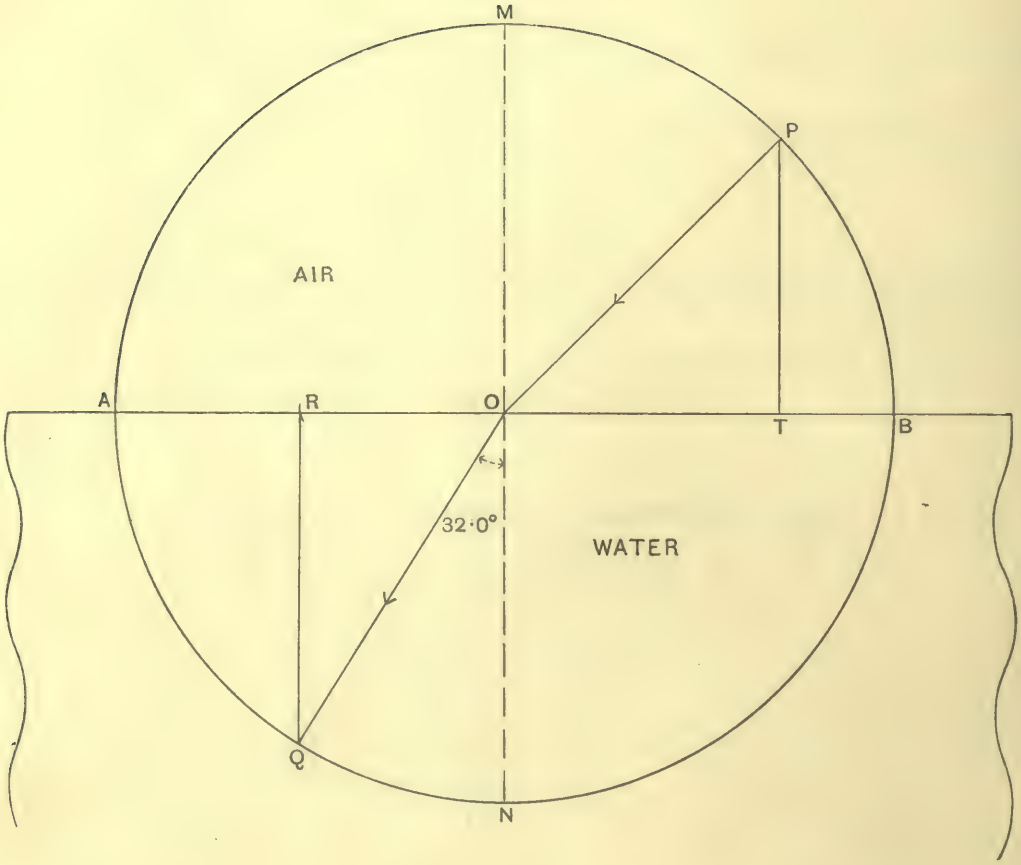
$$= \frac{\sin 70^\circ}{\mu}$$

$$= \frac{\cdot 9397}{2{\cdot}42}$$

$$= \cdot 3883$$

$$= \sin 22{\cdot}8^\circ.$$

Angle of refraction = 22·8°.


M
P
AIR
A
R
O
T
B
32·0°
WATER
Q
N

Alternative method for drawing the refracted ray

To draw a ray from air to water

Let angle of incidence = 45°.

Let PO be a ray incident at O and MON the normal at O. With centre O and *any* radius we described a circle $AMBN$ cutting the incident ray in P. We dropped a perpendicular PT upon AB and measured OT.

$$OT = 4{\cdot}75 \text{ cms.}$$

Then with centre O and radius

$$= \frac{1}{\mu} OT$$
$$= \tfrac{3}{4} \times 4{\cdot}75$$
$$= 3{\cdot}56 \text{ cms.}$$

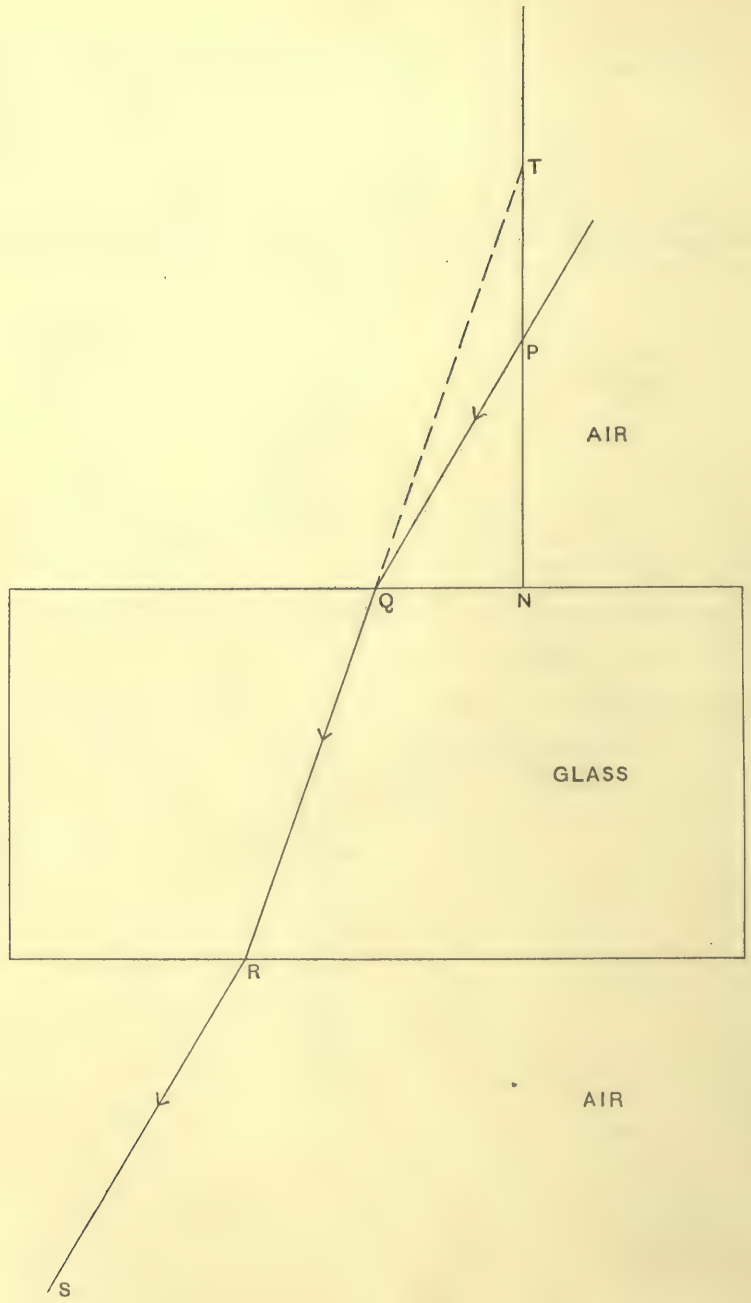
we drew part of a circle cutting OA in R. We drew a normal RQ at R, cutting the circle $AMBN$ in Q, and joined OQ.

Then OQ is the refracted ray. We measured the angle of refraction.

Angle of refraction = 32·0°.

That this construction is correct is clear from the following reasoning :—

$$\frac{\text{sin angle of incidence}}{\text{sin angle of refraction}} = \frac{\sin POM}{\sin QON}$$
$$= \frac{\sin OPT}{\sin OQR}$$
$$= \frac{\dfrac{OT}{OP}}{\dfrac{OR}{OQ}}$$
$$= \frac{OT}{OR}$$
$$= \frac{OT}{\frac{3}{4} OT}$$
$$= \tfrac{4}{3}$$
$$= \mu.$$


T
P
AIR
Q
N
GLASS
R
AIR
S

To find the index of refraction from air to glass

We took a rectangular block of glass, placed it on a sheet of paper on a drawing board and drew a line round it. Against the back edge we placed a pin at Q and another pin at P in such a way that PQ made an angle of $60°$ *with the edge.* Looking through the glass we placed a third pin at R, touching the block, in a line with P and Q, and a fourth at S, away from the block, also in the same straight line.

We removed the block and joined PQ, QR and RS.

From PQ we cut off a length PQ equal to 4 cms. and through P drew a normal to the surface, PN. We produced the ray RQ back to cut the normal in T. We measured QT.

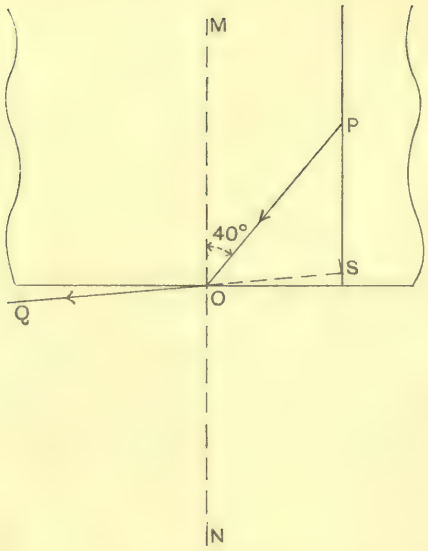
$$QT = 5{\cdot}98 \text{ cms.}$$

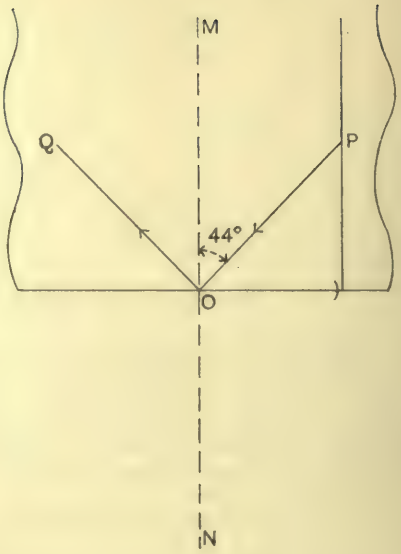
$$\text{Then } {}_A\mu_G = \frac{QT}{QP}$$

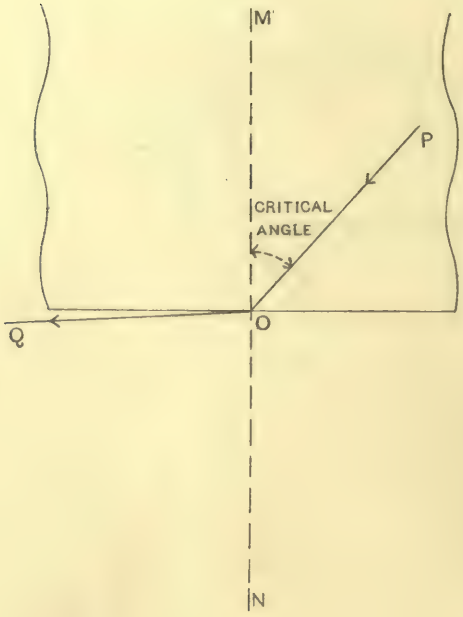
$$= \frac{5{\cdot}98 \text{ cms.}}{4{\cdot}00 \text{ cms.}}$$

$$= 1{\cdot}495.$$

Index of refraction from air to glass = 1·495.


M
P
40°
S
Q
O
N


M
Q
P
44°
O
N


M
P
CRITICAL
ANGLE
Q
O
N

Total Reflection

We drew a ray from glass to air with angles of incidence 40° and 44°. In the first case the ray issues making a small angle *with the surface*, but in the second the construction for finding the refracted ray breaks down, the circle with radius equal to μOP failing to cut the normal. If the angle of incidence is such that the circle *just cuts* the base of the normal, it will be found that the refracted ray just grazes the surface of the glass. The angle of incidence, when this takes place, is called the "critical angle."

Def. Critical Angle.

"If a ray *is* travelling in a medium in such a direction that the emergent ray just grazes the surface of the medium, the angle which *the incident ray* makes with the normal is called the critical angle."

If a ray makes with the normal an angle *greater* than the critical angle, it cannot emerge but is totally reflected, *i.e.* we get "total reflection."

Each medium has its own critical angle.

It is very important to note that total reflection takes place only when a ray is passing *from a denser to a less dense medium.*

To find the critical angle for any two media by calculation

$$\mu = \frac{\text{sin angle of incidence}}{\text{sin angle of refraction}}$$
$$= \frac{\text{sin critical angle}}{\text{sin } 90^\circ}$$
$$= \frac{\text{sin critical angle}}{1}$$
$$= \text{sin critical angle.}$$

$\therefore$ sin critical angle $= \mu$.

To find the critical angle from glass to air

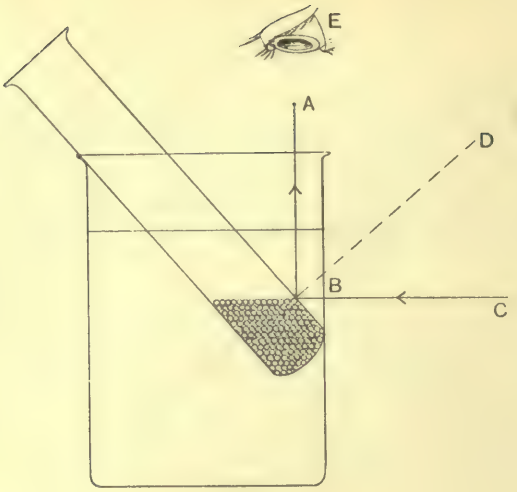
$$\text{sin critical angle} = {}_G\mu_A$$
$$= \tfrac{2}{3}$$
$$= \cdot 666$$
$$= \text{sin } 41^\circ 48',$$

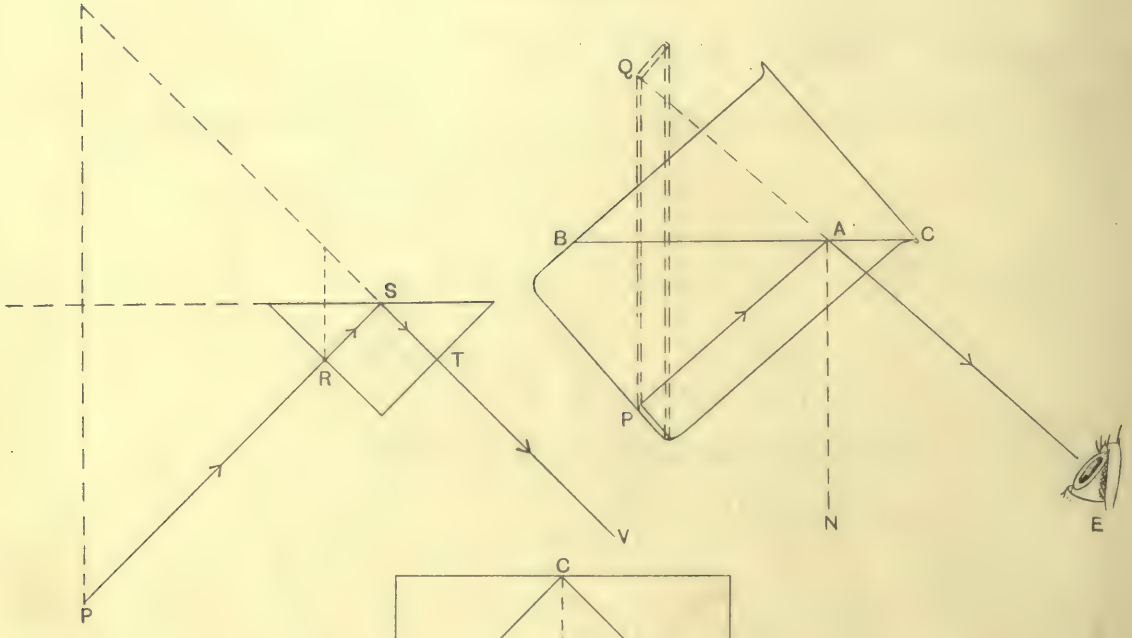
$\therefore$ Critical angle $= 41^\circ 48'$.

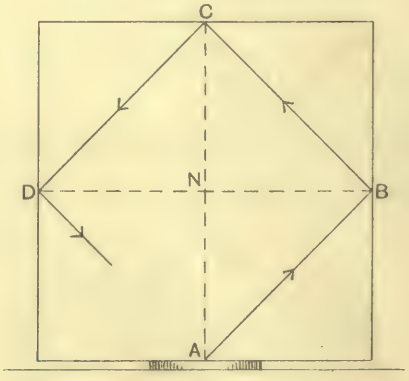
To find the critical angle from water to air

$$\text{sin critical angle} = {}_W\mu_A$$
$$= \tfrac{3}{4}$$
$$= \cdot 75$$
$$= \text{sin } 48^\circ 36',$$

$\therefore$ Critical angle $= 48^\circ 36'$.


E
A
D
B
C


S
R
T
P
V
Q
B
A
C
P
N
E


C
D
N
B
A

Examples of Total Reflection

1. We put some shot into a test-tube and placed it in a beaker of water in a slanting position. On looking down through the water vertically on to the side of the test-tube the shot was not visible, the side of the test-tube having the appearance of a mercury surface. This is due to the light which strikes it being unable to pass *from the water to the air inside the tube*, since the angle of incidence, ABD, is greater than the critical angle for water to air, *i.e.* 48° 36′.

On tilting the test-tube, so that the angle ABD was less, the shot became visible.

2. Total Reflection Prism.

We took a right-angled prism and arranged it as shown in the diagram, placing pins at P and R. Then, looking through the other side, we arranged two more pins, T and V, so that all four appeared to be in the same straight line. The ray PRS, on striking the surface at S, makes an angle of 45° with the normal, which is *greater* than the critical angle for glass to air, *i.e.* 41° 48′. It is therefore totally reflected and is turned through a right angle.

The advantage of a Total Reflection Prism for use as a mirror is that it gives a *single* surface for reflection, and not *two* as in the case of an ordinary silvered mirror, where there is reflection from both the front and back surfaces.

It is used in the following instruments:—

(1) Spectrometer.

(2) Camera obscura.

(3) Prismatic field glasses.

(4) Periscope.

3. E is an eye which observes the coin P by total reflection at the under surface, BC, of some water in a beaker. Q is the virtual image produced.

This occurs when the angle PAN is greater than the critical angle for water to air.

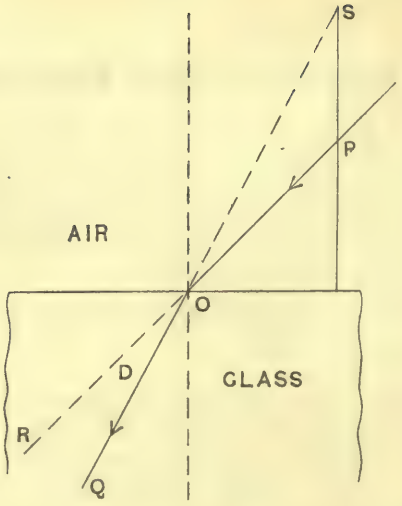
4. Coin beneath a cube of glass.

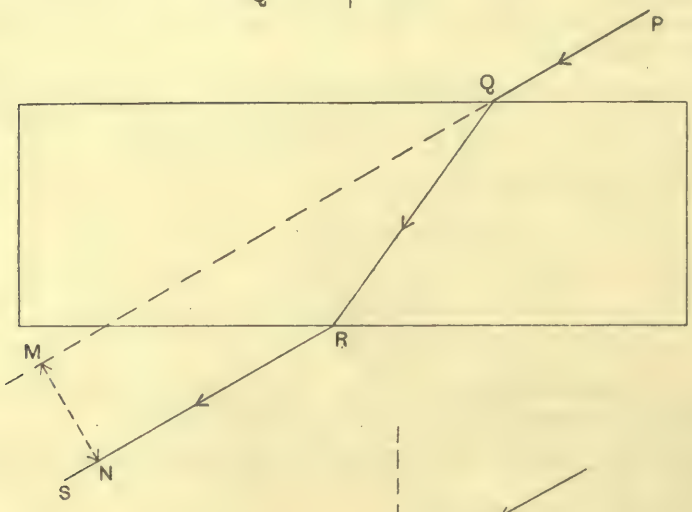
The coin is quite invisible *through the sides of the cube* owing to total reflection.

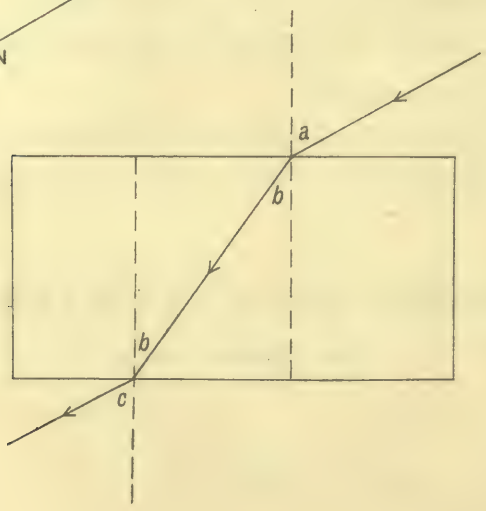
This is clear from the diagram, for, if the angle NAB is not 45° or more, the angle NBA will be.

To find the Refractive Index of a Liquid by Total Reflection

(See advanced notes.)


S
P
AIR
O
D
GLASS
R
Q


P
Q
R
M
S
N


a
b
b
c

Deviation

When light falls obliquely on the surface of a medium of different density and undergoes refraction, the light is said to be "deviated," and the "deviation" is measured by the angle between the refracted ray and the incident ray produced. In the diagram

D = deviation.

To show how deviation varies with the angle of incidence

We drew a ray from air to glass with angles of incidence 30°, 45° and 60°, and measured the deviation in each case.

Angle	Deviation
30°	10°
45°	17°
60°	24°

This shows that, as the angle of incidence increases, the deviation increases. This is what we should expect, since, when the angle of incidence is nothing, *i.e.* when the ray is normal to the surface, there is no deviation, the ray passing straight through.

Refraction through a parallel plate

To prove that when a ray passes through a parallel plate it merely undergoes lateral displacement, *i.e.* the emergent ray is parallel to the incident ray

1. By experiment.

We took a flat glass plate and put pins at P and Q. Looking through the plate we put two more pins at R and S, so that all the four appeared to be in the same straight line.

Lateral displacement = MN.

2. By drawing.

We drew the same thing on the next page. It is clear from the diagram that the emergent ray is parallel to the incident ray produced.

3. By theory.

$$\frac{\sin a}{\sin b} = {}_A\mu_G,$$

$$\frac{\sin c}{\sin b} = {}_A\mu_G.$$

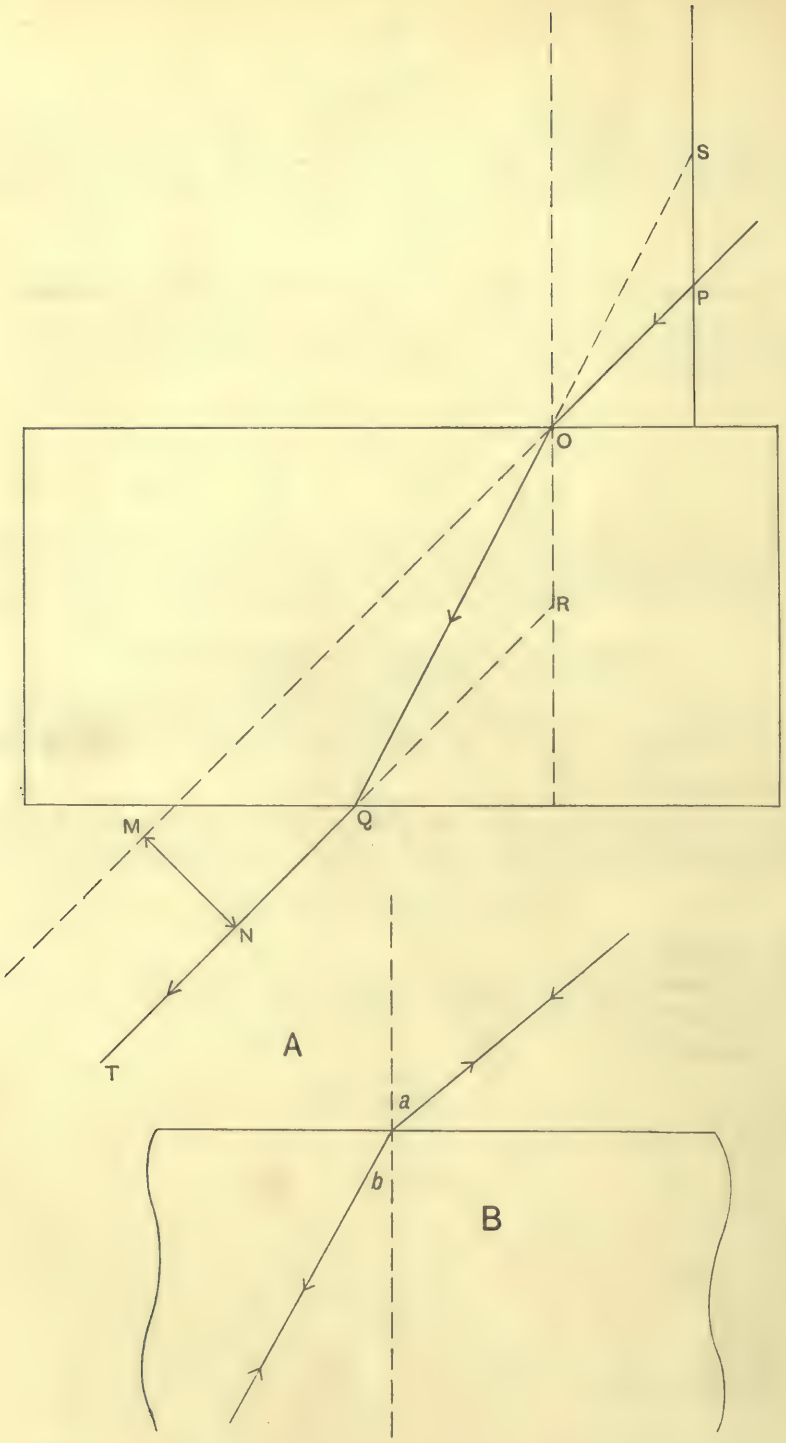
Dividing,

$$\frac{\sin a}{\sin c} = 1,$$

$$\therefore \quad \sin a = \sin c.$$

$$\therefore \quad a = c,$$

i.e. the two rays are parallel.


S
P
O
R
Q
M
N
T
A
a
b
B

To find the lateral displacement produced by a parallel plate

Measure the *perpendicular distance* between the emergent ray and the incident ray produced.

The magnitude of the lateral displacement depends upon three things:—

1. The thickness of the plate.

The greater the thickness of the plate, the greater the lateral displacement.

2. The angle of incidence.

We found the lateral displacement for various angles, taking a plate 5 cms. thick.

Angle	Displacement
30°	1·10 cms.
45°	1·95 cms.
60°	2·65 cms.

The greater the angle of incidence, the greater the lateral displacement.

3. The refractive index.

The greater the refractive index, the greater the lateral displacement.

To show that the index of refraction from a medium A to a medium B is the reciprocal of the index of refraction from a medium B to a medium A

In the diagram,

$$_A\mu_B = \frac{\sin a}{\sin b}.$$

On reversing the ray,

Angle of incidence $= b$.

Angle of refraction $= a$.

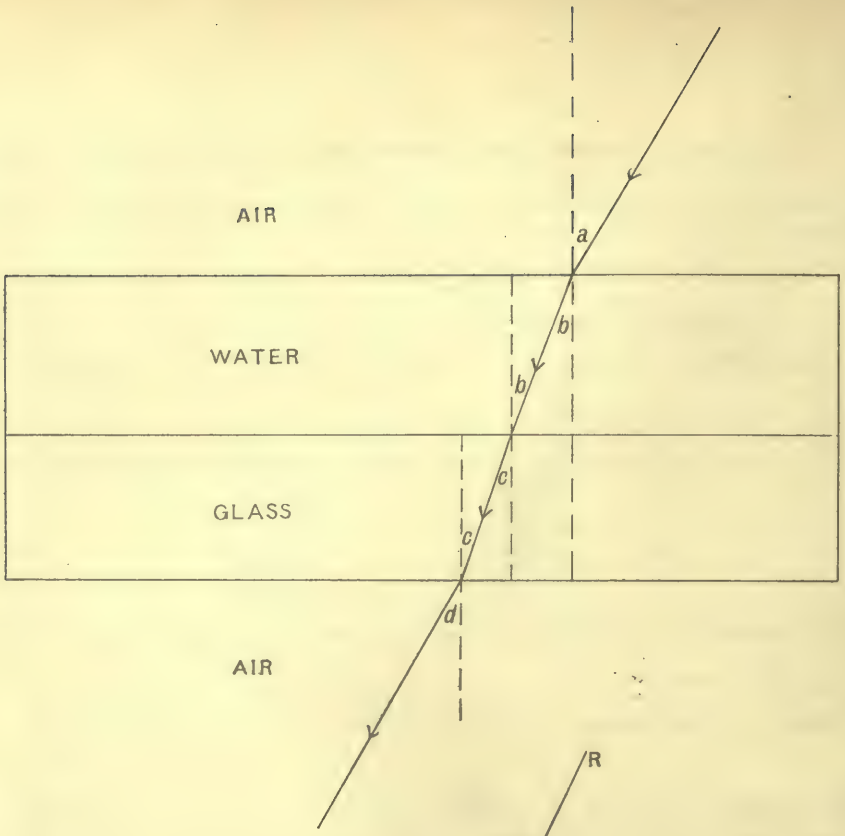
$$_B\mu_A = \frac{\sin b}{\sin a}.$$

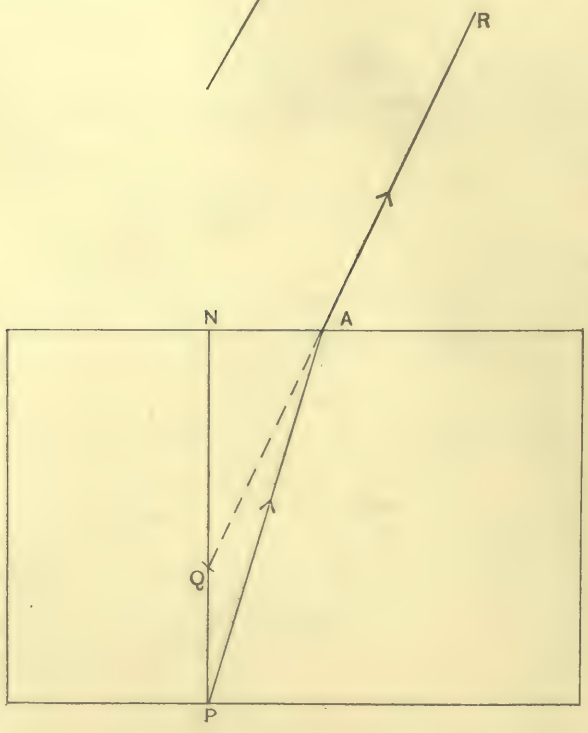
Multiplying,

$$_A\mu_B \times {}_B\mu_A = \frac{\sin a}{\sin b} \times \frac{\sin b}{\sin a}$$

$$= 1.$$

$$\therefore \quad {}_B\mu_A = \frac{1}{{}_A\mu_B}.$$


AIR
a
WATER
b
b
GLASS
c
c
d
AIR


R
N
A
Q
P

To prove that $_A\mu_G = {}_A\mu_W \times {}_W\mu_G$

$$_A\mu_W = \frac{\sin a}{\sin b},$$

$$_W\mu_G = \frac{\sin b}{\sin c},$$

$$_G\mu_A = \frac{\sin c}{\sin d}.$$

But since $a = d$,

$$\sin a = \sin d.$$

$$\therefore\ {}_G\mu_A = \frac{\sin c}{\sin a}.$$

Multiplying,

$$_A\mu_W \times {}_W\mu_G \times {}_G\mu_A = \frac{\sin a}{\sin b} \times \frac{\sin b}{\sin c} \times \frac{\sin c}{\sin a}$$

$$= 1,$$

$$\therefore\ {}_A\mu_W \times {}_W\mu_G = \frac{1}{{}_G\mu_A}$$

$$= {}_A\mu_G.$$

$$\therefore\ {}_A\mu_G = {}_A\mu_W \times {}_W\mu_G.$$

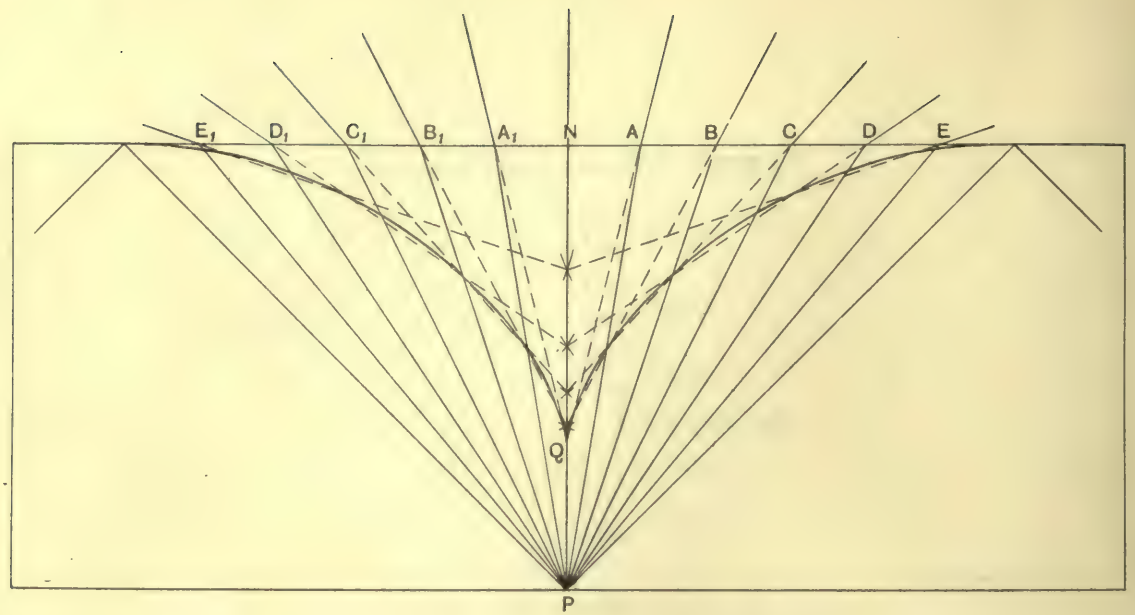
This is a kind of *standard* formula. Thus, if we want to find $_W\mu_G$, we know that

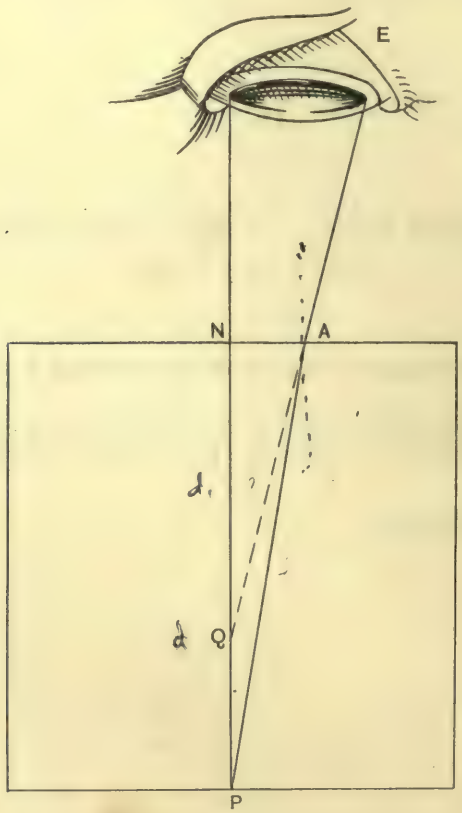
$$_W\mu_G = {}_W\mu_A \times {}_A\mu_G.$$

To draw a ray from glass to air without drawing a special normal

We measured AP and, with centre A and radius $= {}_G\mu_A \,.\, AP$, we drew part of a circle cutting the normal in Q. We joined up QA with a dotted line and produced it onwards to R.

Then AR is the refracted ray.


E1
D1
C1
B1
A1
N
A
B
C
D
E
Q
P


E
N
A
d1
d
Q
P

To find the image of a point formed by refraction in a denser medium

Let P be a luminous point 6 cms. beneath the surface of a block of glass. We drew PN, the normal to the surface at P, and from P drew the rays PA, PB, PC, etc., to meet the surface at the points A, B, C, etc., each 1 cm. apart. We drew the refracted rays with the usual construction. We found that the sixth ray was *totally reflected*.

We found that the refracted rays produced backwards were tangents to a curve, the *caustic curve*, which can be drawn by joining up the points where successive rays meet.

For an eye placed nearly vertically the rays from P appear to come from Q, but, as the eye is moved to the right, the rays appear to come from points situated higher up on the caustic curve. The image of P may be found for any position of the eye by finding where two adjoining rays meet on being produced backwards.

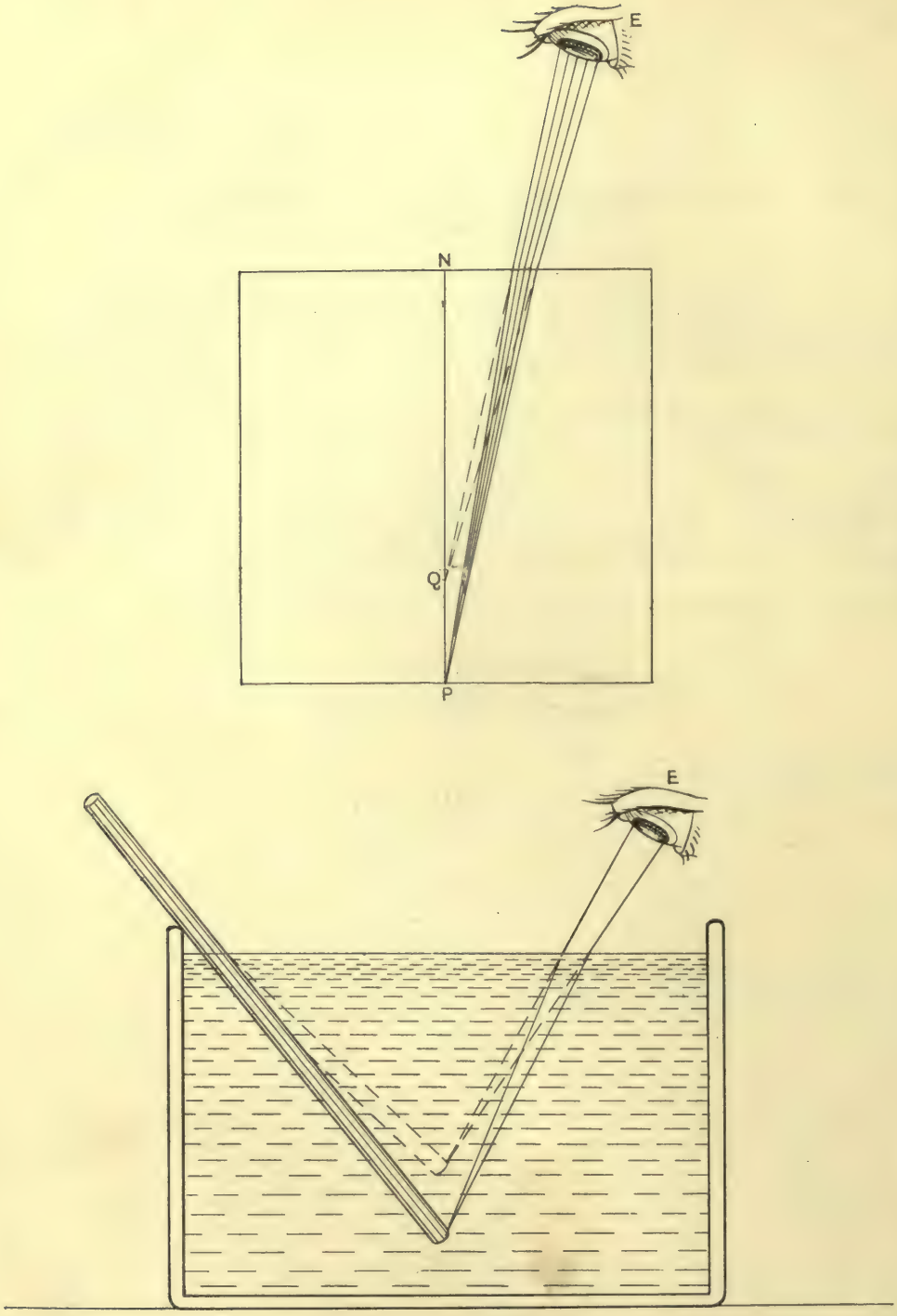
For an eye placed nearly vertically Q is the virtual image of P.

To find the position of Q

When A is near to N,

$$AQ = NQ \text{ nearly,}$$
$$AP = NP \text{ nearly.}$$
$$\text{Now} \quad AQ = {}_G\mu_A . AP.$$
$$\therefore \quad NQ = {}_G\mu_A . NP$$
$$= \tfrac{2}{3} NP.$$

i.e. Q is situated $\frac{1}{3}$ of the way up PN.


E
N
Q'
P
E

To draw the rays by which an eye sees the image of a point in a denser medium

1. If the eye is placed *nearly* vertically.

P is a point beneath the surface of some water.

We drew a normal PN and in it took a point Q such that

$$NQ = \tfrac{3}{4} NP.$$

We drew a cone of rays in the usual way, first to the image and then to the object.

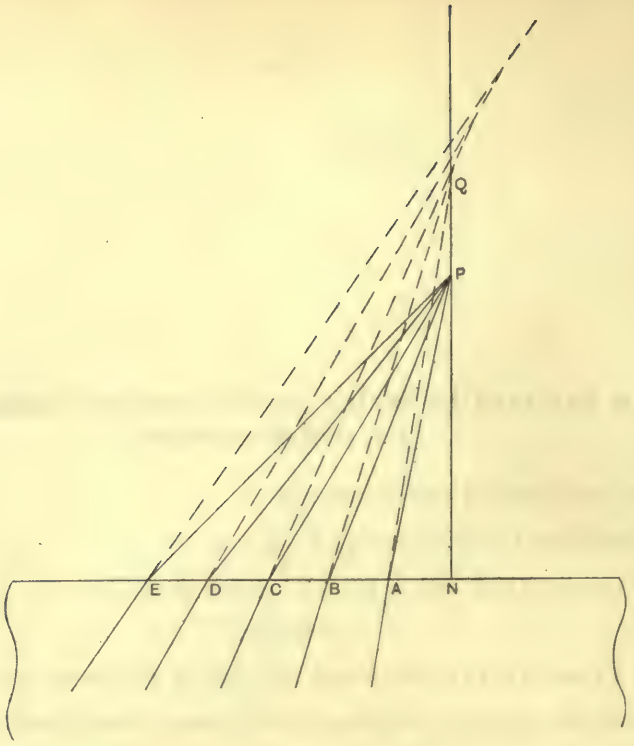
We noticed that the rays on emerging were bent *away from* the normal.

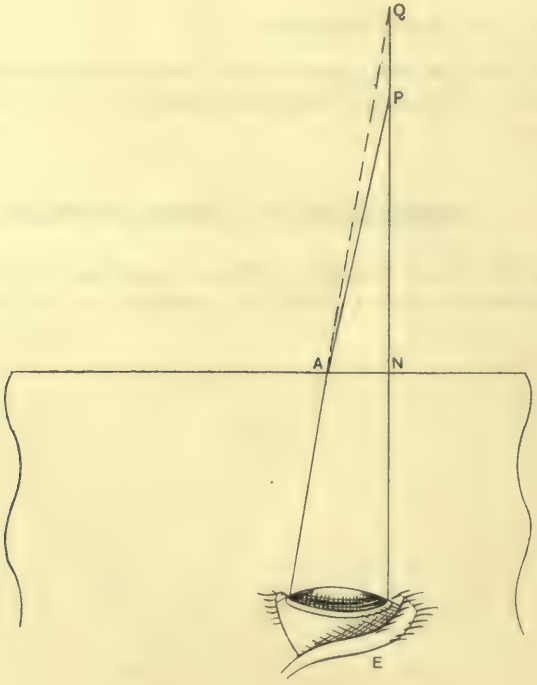
2. If the eye is *not* placed vertically.

It is necessary to draw the caustic curve and then to draw two rays from the eye, tangents to the curve. The point where they meet gives the image Q.

Appearance of a stick in water

The stick must be drawn straight. The end appears to be raised $\frac{1}{4}$ of the depth. We drew the rays from the eye, first to the *apparent* end of the stick and then to the *real* end.


Q
P
E
D
C
B
A
N


Q
P
A
N
E

To find the image of a point formed by refraction in a less dense medium

Let P be a luminous point 5 cms. above the surface of some water. We drew PN, the normal to the surface at P, and from P drew the rays PA, PB, PC, etc., to meet the surface at the points A, B, C, etc., each 1 cm. apart. We drew the refracted rays with the usual construction.

The refracted rays produced backwards are tangents to a curve, the *caustic curve*.

For an eye placed in the denser medium nearly vertically beneath P the rays from P appear to come from Q, but, as the eye is moved to the left, the rays appear to come from points situated higher up on the caustic curve.

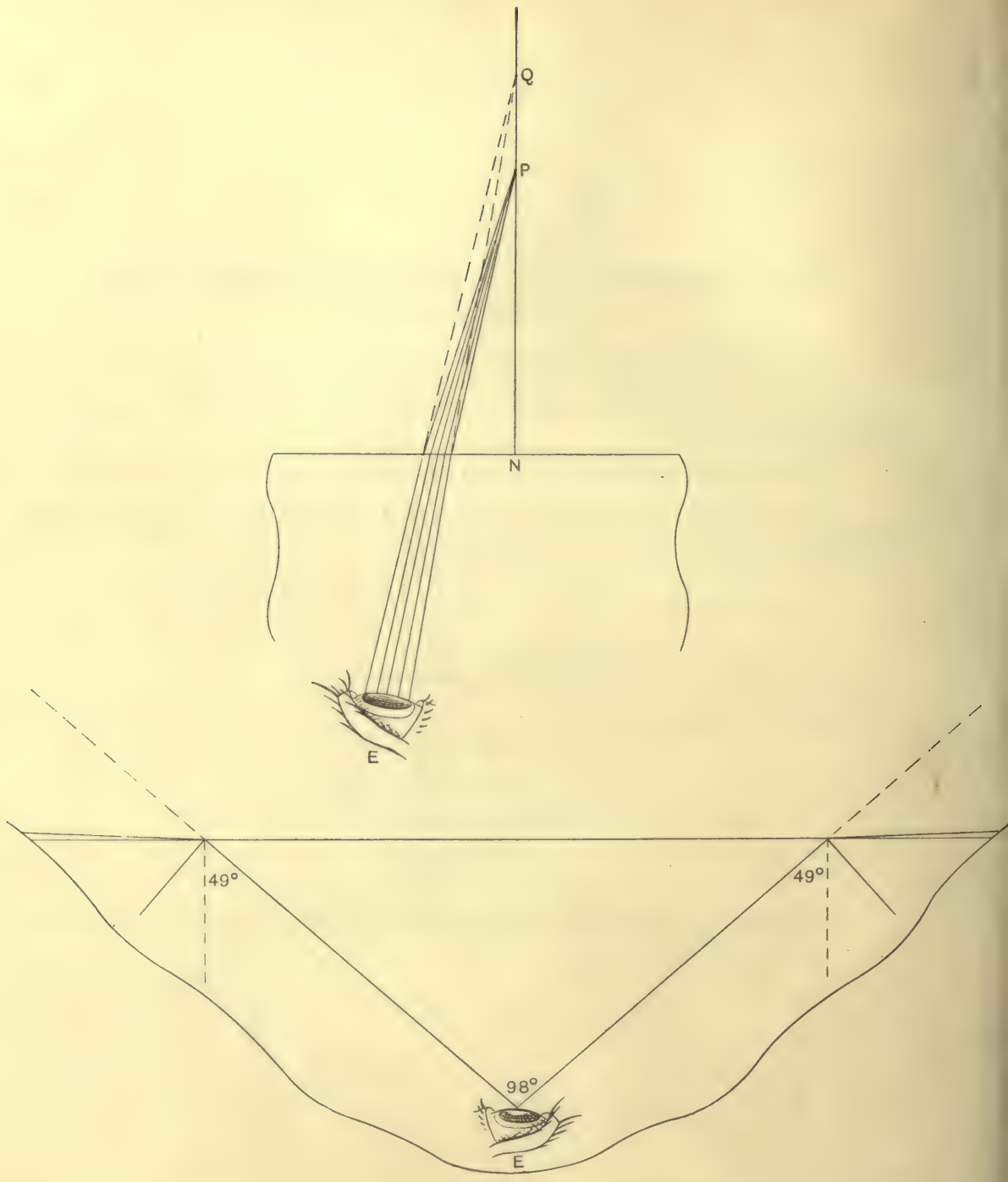
For an eye placed nearly vertically beneath P, Q is the virtual image of P.

To find the position of Q

When A is near to N,

$$AQ = NQ \text{ nearly,}$$
$$AP = NP \text{ nearly.}$$
$$\text{Now} \quad AQ = {}_A\mu_W . AP.$$
$$\therefore \quad NQ = {}_A\mu_W . NP$$
$$= \tfrac{4}{3} NP.$$

i.e. to find Q, draw a normal PN through P and cut off from it a distance $\frac{4}{3}$ of NP.


Q
P
N
E
49°
49°
98°
E

To draw the rays by which an eye in water sees the image of a point in air

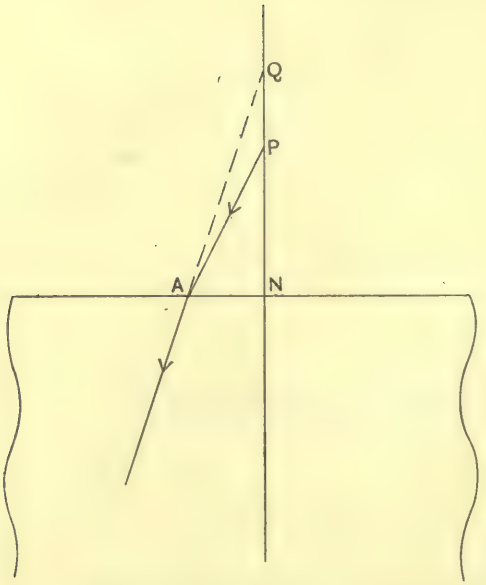
In the diagram

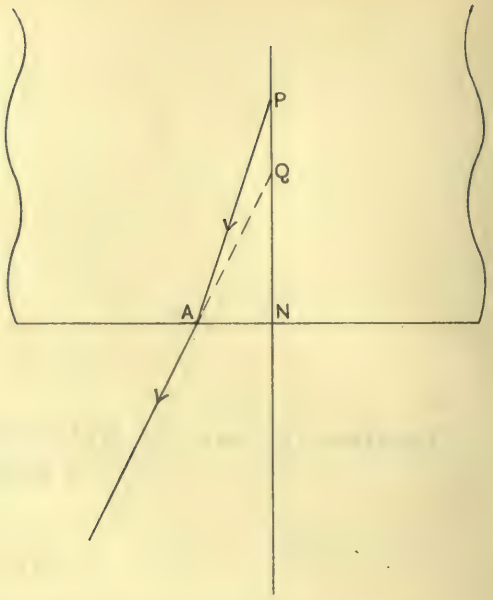
$$NQ = \tfrac{4}{3}\, NP.$$

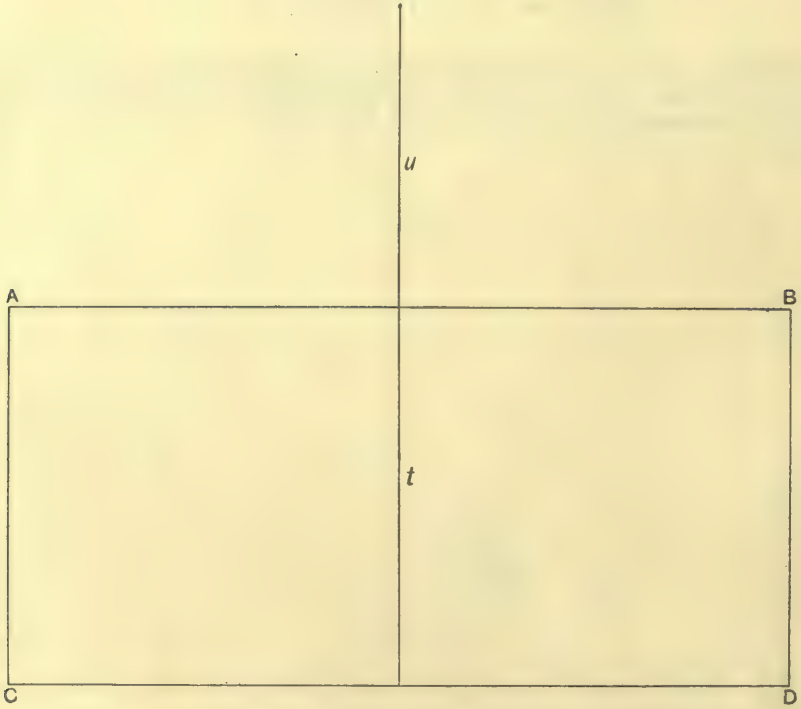
We drew a cone of rays in the usual way, first to the image and then to the object.

What an eye in water sees

Since, when the angle is critical, the emergent ray just grazes the surface of the denser medium, an eye in water will see *the whole of the field*, but it will be compressed into a cone, which is bounded by the rays which are totally reflected. The dotted line shows the boundary of the cone.


Q
P
A
N


P
Q
A
N


u
A
B
t
C
D

To find the image formed by refraction through a transparent plate

The image formed by refraction at a plane surface is obtained as follows :—

Let $_A\mu_M$ = index of refraction from air to medium.

u = distance of object from surface.

v = distance of image from surface.

1. From less dense to denser medium.

$$AQ = {}_A\mu_M \cdot AP.$$

$$\therefore \quad NQ = {}_A\mu_M \cdot NP,$$

$$v = {}_A\mu_M \cdot u.$$

2. From denser to less dense medium.

$$AQ = {}_M\mu_A \cdot AP$$

$$= \frac{AP}{{}_A\mu_M}.$$

$$\therefore \quad NQ = \frac{NP}{{}_A\mu_M},$$

$$v = \frac{u}{{}_A\mu_M}.$$

To find the image formed by refraction through a transparent plate we apply this as follows :—

Let t = thickness of plate.

u = distance of object from first surface, AB.

μ = index of refraction from air to plate.

Then distance of image formed by refraction at first surface, AB, from AB,

$$= \mu u.$$

This image serves as a virtual object for refraction at the second surface, CD.

Distance of virtual object from $CD = \mu u + t$.

Distance of image formed by refraction at second surface, CD, from CD,

$$= \frac{\mu u + t}{\mu}$$

$$= u + \frac{t}{\mu}.$$

u
A
B
t
C
D

Distance of this image from first surface, AB,

$$= u + \frac{t}{\mu} - t.$$

Note that this is *less* than u.

$$\text{Apparent displacement of object} = u - \left(u + \frac{t}{\mu} - t\right)$$

$$= u - u - \frac{t}{\mu} + t$$

$$= t - \frac{t}{\mu}$$

$$= \frac{t\mu - t}{\mu}$$

$$= t\left(\frac{\mu - 1}{\mu}\right).$$

This is a displacement *towards* the plate.

Example

Find the apparent displacement of an object produced by a plate of glass of thickness 1 in.

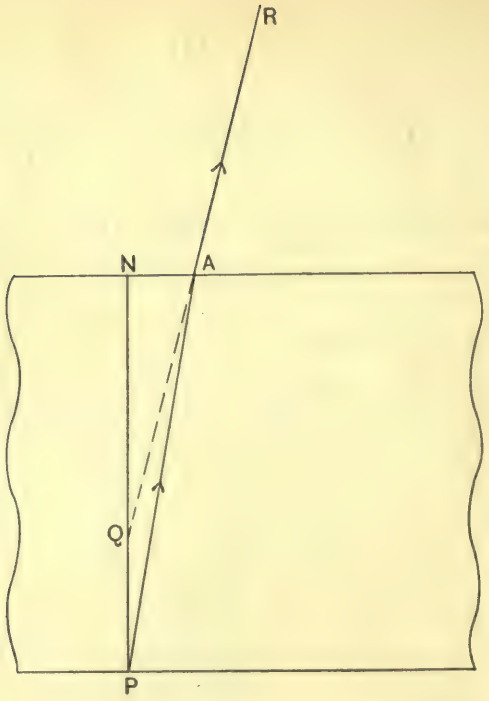
$$\text{Apparent displacement} = t\left(\frac{\mu - 1}{\mu}\right)$$

$$= 1\left(\frac{1{\cdot}50 - 1}{1{\cdot}50}\right)$$

$$= \frac{{\cdot}50}{1{\cdot}50}$$

$$= \tfrac{1}{3} \text{ in.}$$

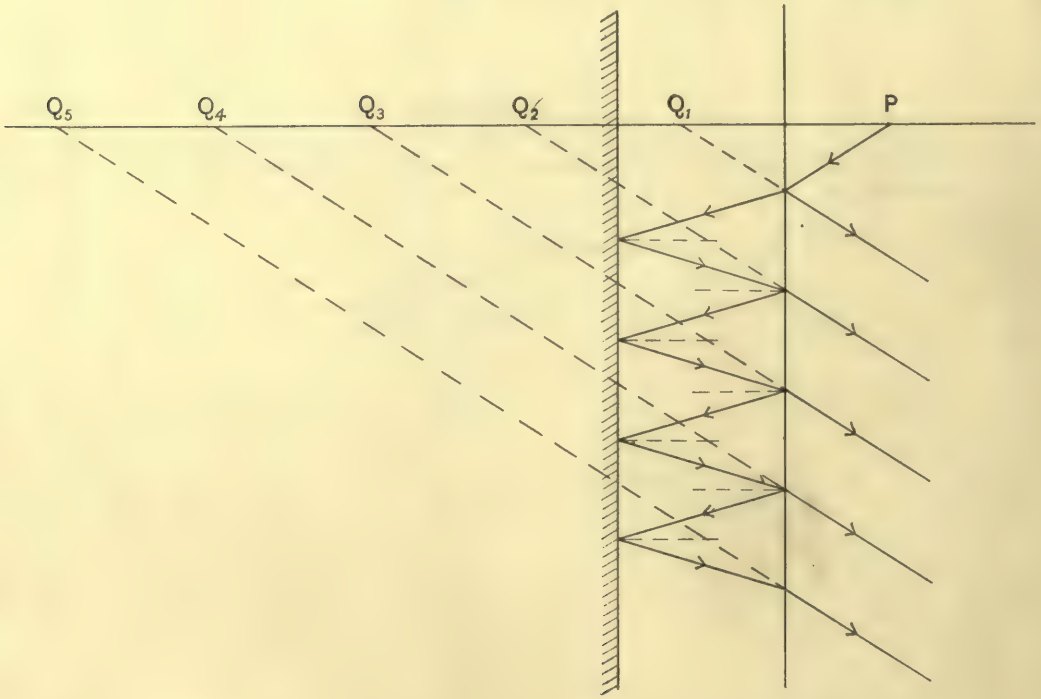
$$\text{Apparent displacement} = \tfrac{1}{3} \text{ in.}$$



2.77

1.68

.98



To find μ by the Microscope

We have seen that, if P is a luminous point beneath a block of glass,

$$AQ = {}_G\mu_A \cdot AP.$$
$$NQ = {}_G\mu_A \cdot NP.$$
$$\therefore {}_G\mu_A = \frac{NQ}{NP},$$
$${}_A\mu_G = \frac{NP}{NQ}.$$

We took a "reading microscope," which carries a scale and a vernier by which its position can be accurately read. We placed a piece of paper, with a pencil mark upon it, on the tray, focussed the microscope upon it and took the reading,

$$P = \cdot 98 \text{ cm.}$$

We then placed a block of glass on top of the paper, focussed, and again took the reading,

$$Q = 1 \cdot 60 \text{ cms.}$$

Then, placing the paper on top of the block, we obtained N,

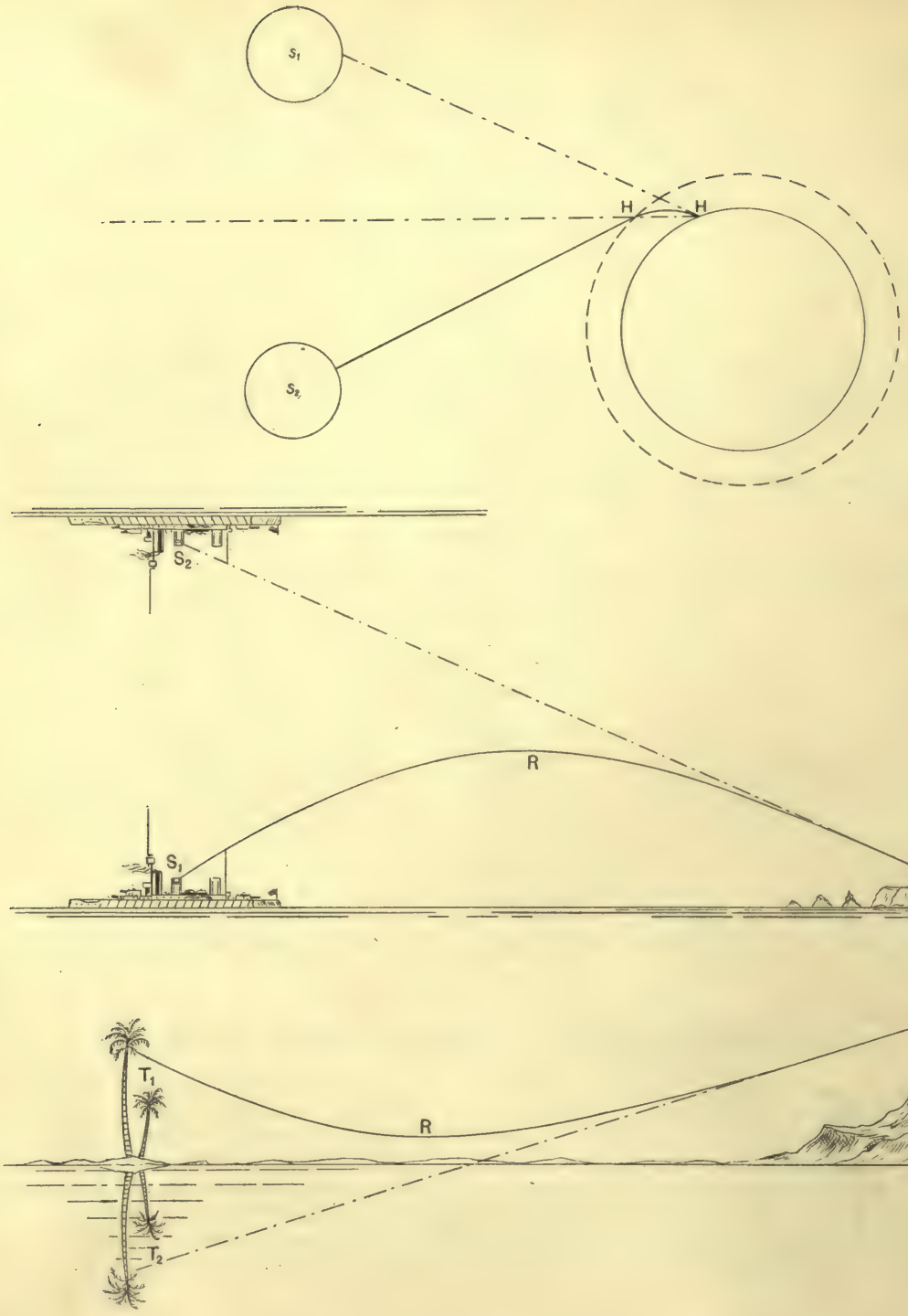
$$N = 2 \cdot 77 \text{ cms.}$$

$$NQ = 2 \cdot 77 - 1 \cdot 60$$
$$= 1 \cdot 17 \text{ cms.}$$
$$NP = 2 \cdot 77 - \cdot 98$$
$$= 1 \cdot 79 \text{ cms.}$$
$$\therefore {}_A\mu_G = \frac{NP}{NQ}$$
$$= \frac{1 \cdot 79 \text{ cms.}}{1 \cdot 17 \text{ cms.}}$$
$$= 1 \cdot 53.$$

This method may also be used for water. To get the surface of the water lycopodium powder is used.

Formation of a number of images by a thick mirror

We placed a candle in front of a *thick* mirror and looked at it from the side. We saw a number of images, Q_1, Q_2, Q_3, etc., of which Q_2 was the brightest. The first image is formed by reflection at the first surface. The second is due to light passing through the glass and being reflected at the back or silvered surface. The light is refracted on entering the glass and also on issuing. Most of the light enters the glass in this way and the second image, Q_2, is consequently the brightest. Some of the light is further reflected back from the front surface and issues after two or more reflections. Thus we get a further series of images, Q_3, Q_4, Q_5, etc.


S1
H
H
S2
S2
R
S1
E
E
T1
R
T2

Refraction by gases

Gases, like liquids and solids, can refract light, but their refractive powers are comparatively feeble. Refraction in gases takes place when light passes from a layer of gas into another layer of the same gas which differs in density. This difference in density is usually produced by difference of temperature.

Examples of Refraction by gases

1. The apparent quivering of objects seen above a hot surface, *e.g.* stones on a sea shore, which is commonly explained as "heat rising," is really due to the irregular refraction of the light as it passes through layers of different density.

2. The sun may sometimes be seen, when it is in reality beneath the horizon, *e.g.* in the Arctic Regions. This is due to the light being refracted towards the normal as it passes from the vacuous space between the earth and the sun into our atmosphere.

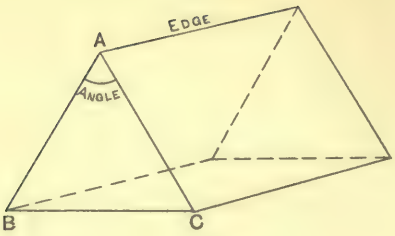
The sun, S_1, which is below the horizon, HH, appears to be at S_2 to an eye placed at H. This makes the sun appear to rise earlier and set later, thus lengthening the day.

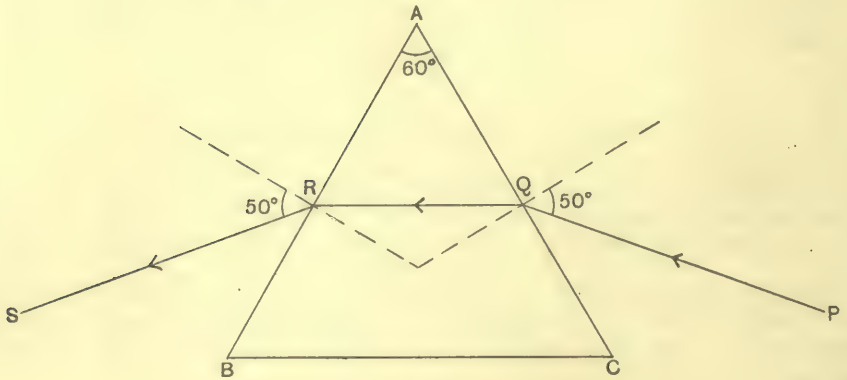
3. Mirage at sea.

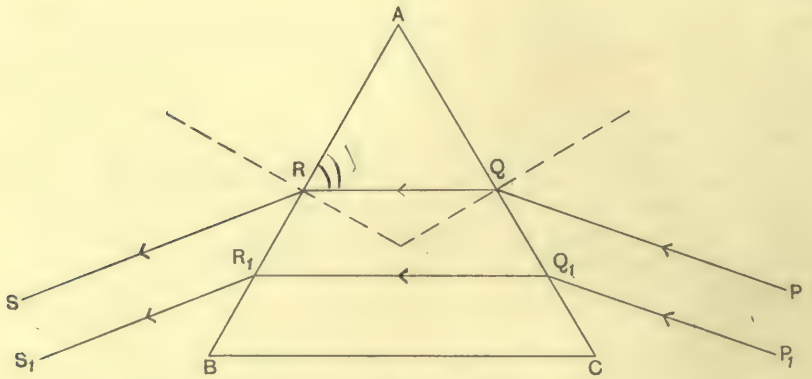
The layers near the sea are colder and therefore denser. The rays from the ship, S_1, are bent *away from* the normal as they pass upwards and are *totally reflected* at R. They are then bent *towards* the normal as they pass downwards and, to an eye at E, appear to come from S_2. This gives an inverted image of the ship suspended in mid-air.

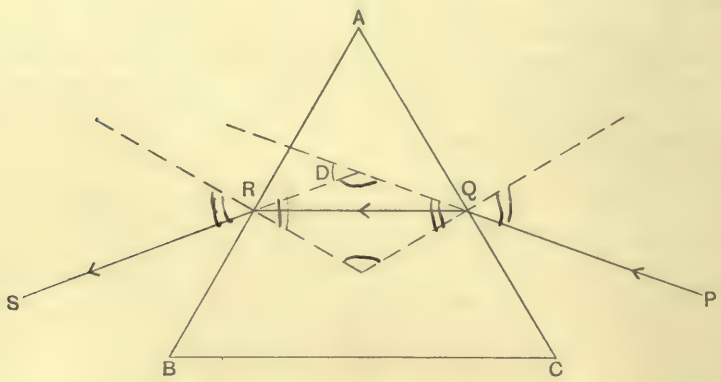
4. Mirage in deserts.

The layers near the surface are warmer and therefore less dense. The rays from a tree, T_1, are bent *away from* the normal as they pass downwards and are *totally reflected* at R. They are then bent *towards* the normal as they pass upwards and, to an eye at E, appear to come from T_2, which gives an inverted image of the tree. This gives the impression to an observer that it is a tree standing near some water in which it is reflected.


A
EDGE
ANGLE
B
C


A
60°
R
Q
50°
50°
S
P
B
C


A
R
Q
R1
Q1
S
S1
P
P1
B
C


A
D
R
Q
S
P
B
C

PRISMS

Def.

"A prism is a portion of a medium having two plane faces which are inclined to one another at an angle."

"The angle of a prism is the angle between the two plane faces."

"The edge of a prism is the line along which the two plane faces meet."

To find the action of a prism upon a ray of light

We arranged a prism upon a piece of paper on a drawing board, drew a line round it, and put in two pins at P and Q, so that PQ made an angle of 50° with the normal to the surface at Q. Looking through the prism we placed two more pins at R and S, so that they appeared to be in the same straight line as P and Q.

We concluded from the experiment that, when a ray of light passes through a prism, it is bent *away from the edge of the prism.* A prism therefore produces *deviation.*

This is true in all cases whatever the angle of incidence may be, unless the ray is totally reflected.

To show that two rays which are parallel before they meet a prism continue parallel after refraction

We traced the course of two rays through a prism by means of pins. This is also obvious by theory.

To show how the deviation varies with the angle of the prism

We took two prisms of different angle and traced the course of a ray through them by means of pins, making the angle of incidence 50° in each case. We measured the deviations.

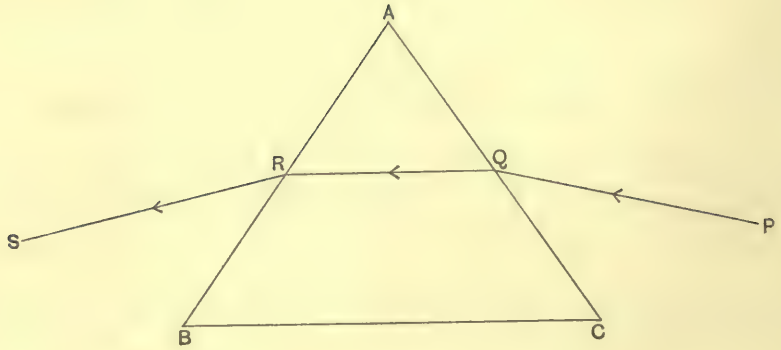
Prism of small angle – Deviation = 18°.
Prism of large angle – Deviation = 33°.

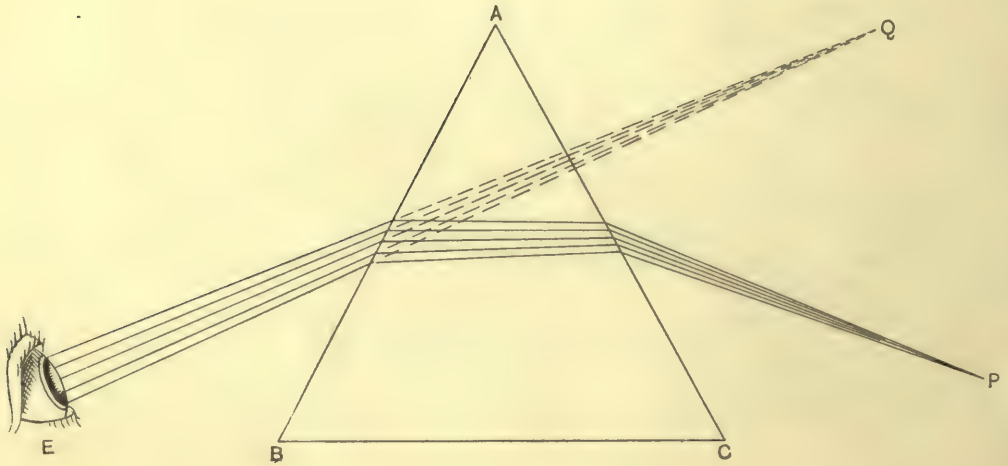
This shows that the greater the angle of the prism the greater the deviation.

Minimum Deviation

The amount of deviation produced by a prism depends upon the angle which the incident ray makes with the normal to the first surface.

Experiments show that, with a given prism, there is a certain value of the angle of incidence for which the deviation is a minimum. The smallest value that the deviation can have is called the angle of "minimum deviation."


A
R
Q
S
P
B
C


A
Q
P
E
B
C

To show that when a ray suffers minimum deviation angle of incidence = angle of emergence

We placed a piece of paper upon a drawing board and drew a horizontal line upon it. We fastened to one of the *triangular* faces of a prism the flat head of a *drawing* pin, made a small vertical hole in the drawing board through a point on the line that we had drawn and inserted the shank of the pin. The prism could rotate freely about the pin.

We then placed two pins, P and Q, along the line on one side of the prism and, looking through the prism from the other side, placed two more pins, R and S, in such a position that they appeared to be in the same straight line as P and Q. We drew the emergent ray and removed the pins R and S.

We then rotated the prism through a small angle and obtained once more the emergent ray. We noted the change in the angle of deviation. We continued to rotate the prism through small angles *in the same direction* and found that, for one particular position of the prism, the deviation was a minimum. We marked the positions of the incident and emergent rays for this position, drew a line round the prism and, after removing it, marked the direction of the ray inside the prism.

We measured the angles of incidence and emergence.

Angle of incidence $= 47°$.

Angle of emergence $= 48°$.

The experiment shows that, when a ray suffers minimum deviation,

angle of incidence = angle of emergence.

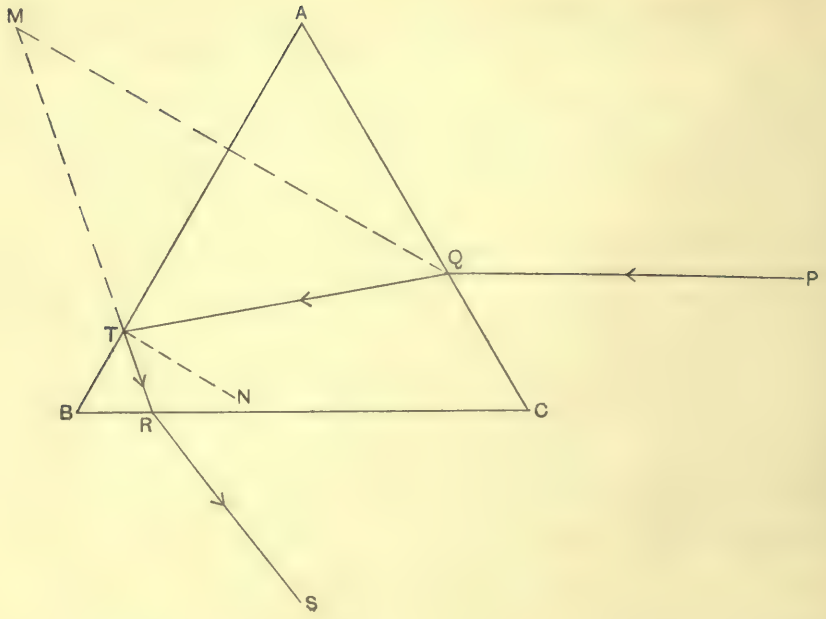
It is easy to show that this must be so, by assuming that they are unequal and reversing the ray. This leads to the conclusion that there must be *two* angles of incidence, which give rise to minimum deviation. This is contrary to experience.

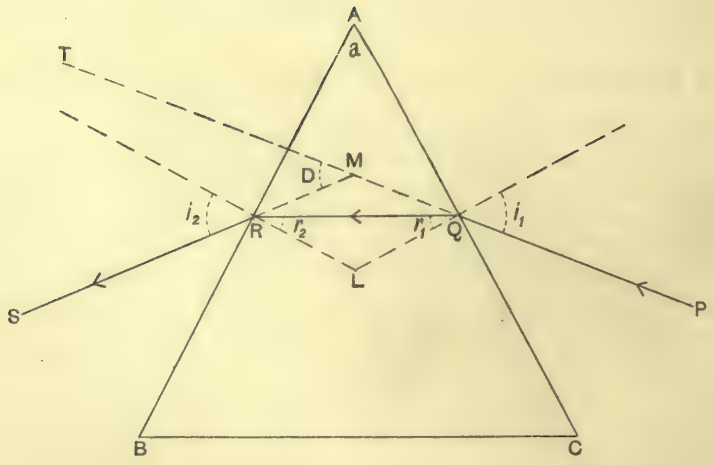
To find the position of the image of a pin seen by refraction through a prism

If a narrow divergent pencil is refracted through a prism so that the central ray of the pencil follows the path of minimum deviation, then the emergent pencil will *appear to diverge from a point*, which is the virtual image. This is so because the angles of incidence of the extreme rays of the pencil differ but slightly from the angle of incidence of the central ray and *the deviation changes but slowly with the angle of incidence, when this is nearly equal to that which corresponds to minimum deviation.*

It follows from this that an object seen through a prism is most distinct when the central ray of the pencil meeting the eye has followed the path of minimum deviation. For other paths the extreme rays of the pencil are deviated by different amounts, the ray on one side of the pencil being deviated *more*, and that on the other *less*, than the central ray, and on emergence the rays do *not* appear to diverge from a single point.

We arranged a prism with some pins, so that a ray passed through in the position of minimum deviation. Looking through the prism we placed a *tall* pin on the other side, so that it coincided with the virtual image, Q, of the object, P, seen through the prism.


M
A
Q
P
T
N
B
R
C
S


A
a
T
M
D
i_2
R
r_2
r_1
Q
i_1
S
L
P
B
C

To find the index of refraction from glass to air by total reflection with a prism

We drew a line upon a piece of paper on a drawing board and placed two pins, P and Q, at the ends of the line. We arranged a prism, ABC, in such a way that the emergent ray just grazed the surface AB. Then, turning the prism slightly, we obtained total reflection at the surface AB, the ray emerging in the direction RS.

To draw the course of the ray inside the prism we found the image, M, of Q and joined MR, cutting the face of the prism AB in T. We joined QT and drew the normal TN at T. We measured the angle QTN, which is the critical angle for glass to air.

$$\angle QTN = 40°.$$

Then $$_G\mu_A = \text{sin critical angle}$$
$$= \sin QTN$$
$$= \sin 40°$$
$$= \cdot 643.$$

Index of refraction from glass to air $= \cdot 643$.

To find the Refractive Index of a Prism

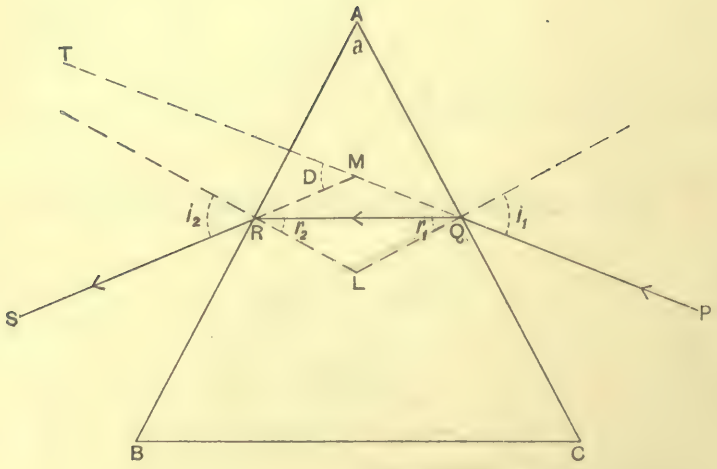
Let
D = deviation.
a = angle of prism.
i_1 = 1st angle of incidence.
r_1 = angle of refraction.
r_2 = 2nd angle of incidence.
i_2 = angle of emergence.

(1) Since QL and RL are normals,

$$\angle AQL + \angle ARL = 180°.$$
$$\therefore \angle QAR + \angle QLR = 180°.$$
$$i.e. \quad a + \angle QLR = 180°.$$

Again
$$r_1 + r_2 + \angle QLR = 180°.$$
$$\therefore r_1 + r_2 = a.$$

(2)
$$\text{Deviation} = D$$
$$= \angle TMR$$
$$= \angle MQR + \angle MRQ$$
$$= (i_1 - r_1) + (i_2 - r_2)$$
$$= i_1 + i_2 - (r_1 + r_2)$$
$$= i_1 + i_2 - a.$$
$$i.e. \quad D = i_1 + i_2 - a.$$
$$\therefore i_1 + i_2 = D + a.$$


A
a
T
M
D
i_2
R
r_2
r_1
Q
i_1
L
S
P
B
C

Now, when the deviation is a minimum,

$$i_1 = i_2,$$

$$r_1 = r_2.$$

But we have seen that

$$i_1 + i_2 = D + a.$$

$$\therefore\ 2i_1 = D + a.$$

$$\therefore\ i_1 = \frac{D+a}{2}.$$

Again $r_1 + r_2 = a.$

$$\therefore\ 2r_1 = a.$$

$$\therefore\ r_1 = \frac{a}{2}.$$

Now $$\mu = \frac{\sin i_1}{\sin r_1}$$

$$= \frac{\sin \frac{D+a}{2}}{\sin \frac{a}{2}}.$$

The value of μ can be found by means of an instrument called the Spectrometer.

The Spectrometer

This consists of the following parts:—

1. A graduated circle supported in a horizontal position upon a suitable stand.

2. A collimator, rigidly attached to the circle, consisting of a tube with a vertical slit at one end and a convex lens at the other, the length of the tube being equal to the focal length of the lens. Rays diverging from the slit are therefore parallel after passing through the lens.

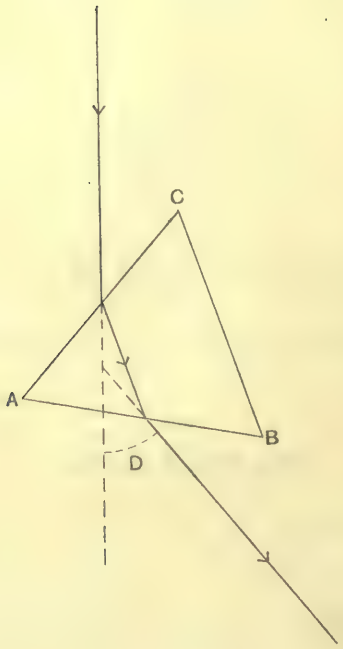
3. A movable arm, which carries a telescope, provided with an eye-piece and cross-wires. The position of the arm can be read by a vernier.

4. A small turn-table arranged above the centre of the circle, on which the prism is fixed.

The Spectrometer is used

(1) For finding the angle and refractive index of a prism.

(2) As a Spectroscope.


C
A
B
D

Adjustment of the Spectrometer

There are three adjustments to be made:—

1. Eye-piece.

Point the telescope to the sky and move the eye-piece lens in or out until the cross-wires are distinctly seen.

2. Telescope.

Fix the telescope in a clamp and turn it towards some distant object. Then, by means of the milled head, adjust the eye-piece and cross-wires until the image of the distant object and the cross-wires coincide. The cross-wires are then in the "focal plane" of the telescope.

3. Collimator.

Illuminate the slit by placing a sodium flame in front of it and arrange the telescope so that it and the collimator tube are in the same straight line. Then, while looking through the telescope, move the slit in or out until the image of the slit and the cross-wires coincide. The slit is then in the "focal plane" of the collimator lens.

To show the phenomenon of Minimum Deviation with a Spectrometer

We illuminated the slit and arranged the prism and telescope so that, on looking through the telescope, we could see the slit by refraction through the prism. We found that, at first, on turning the edge of the prism *to the left*, the slit travelled *to the left* but, later on, it turned and travelled *to the right*. The prism is in the position of minimum deviation at the point where the turning back takes place.

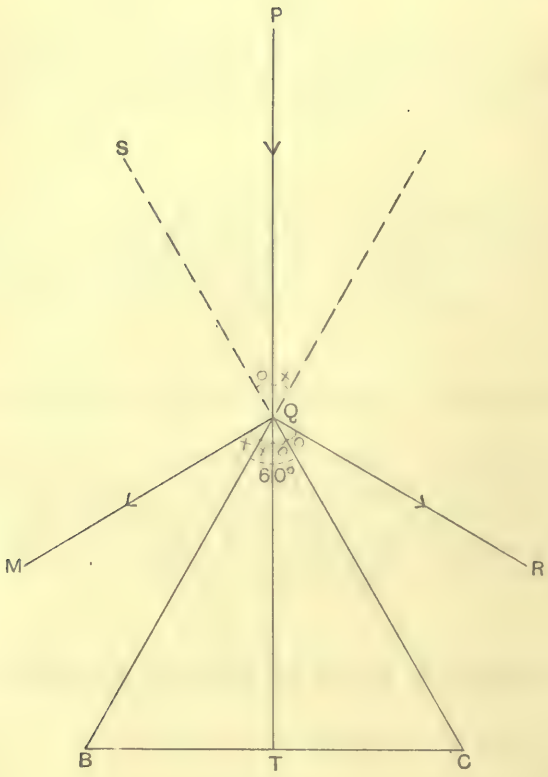
To find the refractive index of glass by the Spectrometer

The experiment consists of two parts:—

1. To find the Deviation when it is Minimum.

We observed the slit directly, after removing the prism, and then through the prism in the position of Minimum Deviation. The difference between the two readings gives the Deviation.

1st Reading = 203° 3′
2nd Reading = 145° 49′
Deviation = 57° 14′


P
S
Q
60°
M
R
B
T
C

2. To find the Angle of the Prism.

We pointed the prism with its edge towards the collimator, so that the light was reflected from both sides of the prism.

$$\begin{aligned} \text{Now} \quad & \angle PQS = \angle RQC, \\ & \angle PQS = \angle TQC. \\ \therefore \quad & \angle TQC = \angle RQC, \\ \text{or} \quad & \angle TQR = 2\angle TQC. \end{aligned}$$

In the same way

$$\begin{aligned} & \angle TQM = 2\angle TQB. \\ \therefore \quad & \angle MQR = 2\angle BQC, \\ \text{or} \quad & \angle BQC = \tfrac{1}{2}\angle MQR. \end{aligned}$$

We measured the angle MQR by the Spectrometer and so found the angle of the prism.

$$\begin{aligned} \text{1st Reading} &= 269^\circ\ 33' \\ \text{2nd Reading} &= 149^\circ\ \ 8' \\ \text{Difference} &= 120^\circ\ 25' \\ \text{Angle of prism} &= \frac{120^\circ\ 25'}{2} \\ &= 60^\circ\ 12'. \end{aligned}$$

$$\begin{aligned} D + a &= 57^\circ\ 14' + 60^\circ\ 12' \\ &= 117^\circ\ 26'. \\ \text{Now} \quad \mu &= \frac{\sin i_1}{\sin r_1} \\ &= \frac{\sin \dfrac{D+a}{2}}{\sin \dfrac{a}{2}} \\ &= \frac{\sin \dfrac{117^\circ\ 26'}{2}}{\sin \dfrac{60^\circ\ 12'}{2}} \\ &= \frac{\sin 58^\circ\ 43'}{\sin 30^\circ\ 6'} \\ &= \frac{\cdot 8545}{\cdot 5015} \\ &= 1\cdot 70. \end{aligned}$$

Refractive index of glass $= 1\cdot 70$.

The glass of which the prism belonging to a Spectrometer is made is exceptionally dense and has an unusually high refractive index.

To calculate the Minimum Deviation for a glass prism of angle 60°

$$\text{Let} \quad \mu = 1{\cdot}50.$$

$$\text{Now} \quad \frac{\sin\frac{D+a}{2}}{\sin\frac{a}{2}} = \mu.$$

$$\therefore \sin\frac{D+a}{2} = \mu \times \sin\frac{a}{2}$$
$$= 1{\cdot}50 \times \sin 30$$
$$= 1{\cdot}50 \times \tfrac{1}{2}$$
$$= {\cdot}75$$
$$= \sin 48{\cdot}6°.$$

$$\therefore \frac{D+a}{2} = 48{\cdot}6°.$$

$$\therefore D + a = 48{\cdot}6 \times 2$$
$$= 97{\cdot}2°.$$

$$\therefore D = 97{\cdot}2 - a$$
$$= 97{\cdot}2 - 60$$
$$= 37{\cdot}2°.$$

$$\text{Minimum Deviation} = 37{\cdot}2°.$$

To find the angle of incidence which will give Minimum Deviation

$$\text{Deviation at first surface} = \frac{37{\cdot}2}{2}$$
$$= 18{\cdot}6°.$$

$$\text{Let} \quad x = \text{angle of incidence.}$$

$$\text{Then} \quad \frac{\sin x}{\sin(x - 18{\cdot}6)} = 1{\cdot}50,$$

$$\sin x = 1{\cdot}50 \times \sin(x - 18{\cdot}6),$$

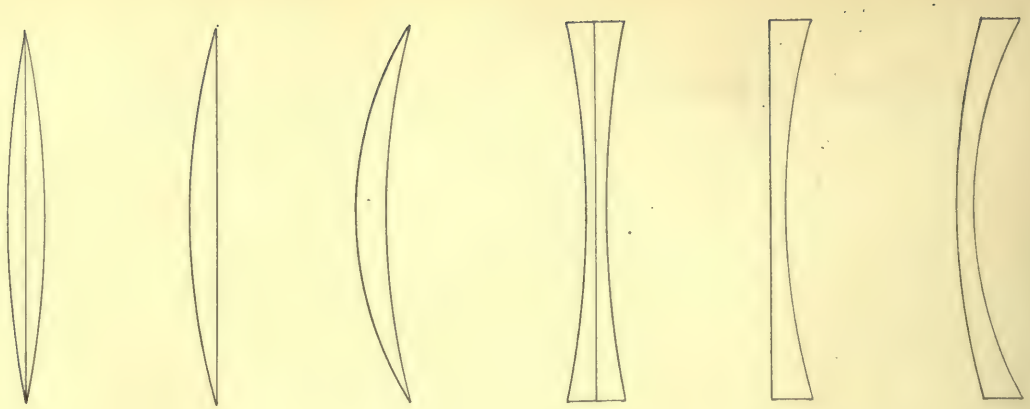
$$\sin x = \tfrac{3}{2}(\sin x \cos 18{\cdot}6 - \cos x \sin 18{\cdot}6),$$

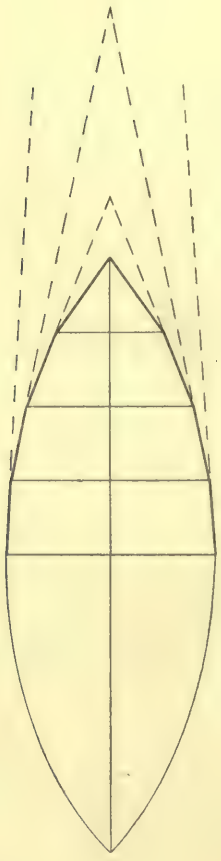
$$\sin x\,(1 - \tfrac{3}{2}\cos 18{\cdot}6) = -\tfrac{3}{2}\cos x \sin 18{\cdot}6.$$

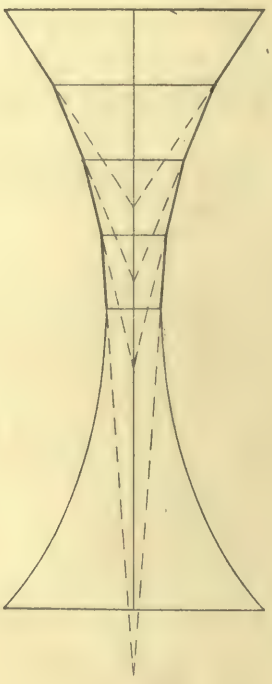
$$\therefore \tan x = \frac{-\frac{3}{2}\sin 18{\cdot}6}{1 - \frac{3}{2}\cos 18{\cdot}6}$$
$$= 1{\cdot}135.$$

$$\therefore x = 48{\cdot}6°.$$

$$\text{Angle of incidence} = 48{\cdot}6°.$$







LENSES

A lens is usually a piece of glass bounded by two surfaces which are parts of spheres.

There are two principal classes of lenses:—

1. Converging or Convex Lenses.
   - (1) Double convex.
   - (2) Plano-convex.
   - (3) Concavo-convex or convex meniscus.

They are *thicker* in the middle than at the edges.

They are called *converging* because the rays are always more *convergent* after passing through them.

2. Diverging or Concave Lenses.
   - (1) Double concave.
   - (2) Plano-concave.
   - (3) Convexo-concave or concave meniscus.

They are *thinner* in the middle than at the edges.

They are called *diverging* because the rays are always more *divergent* after passing through them.

Def.

"The principal axis of a lens is the line joining the centres of the two spheres, parts of which go to form the lens."

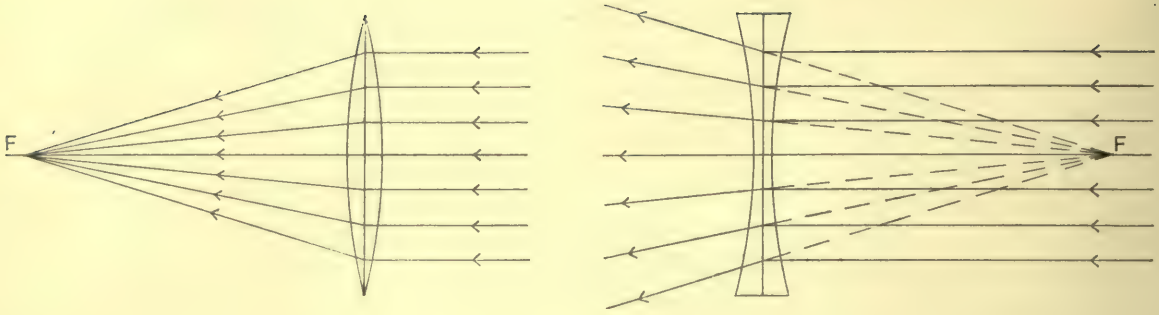
Idea of Collection of Prisms

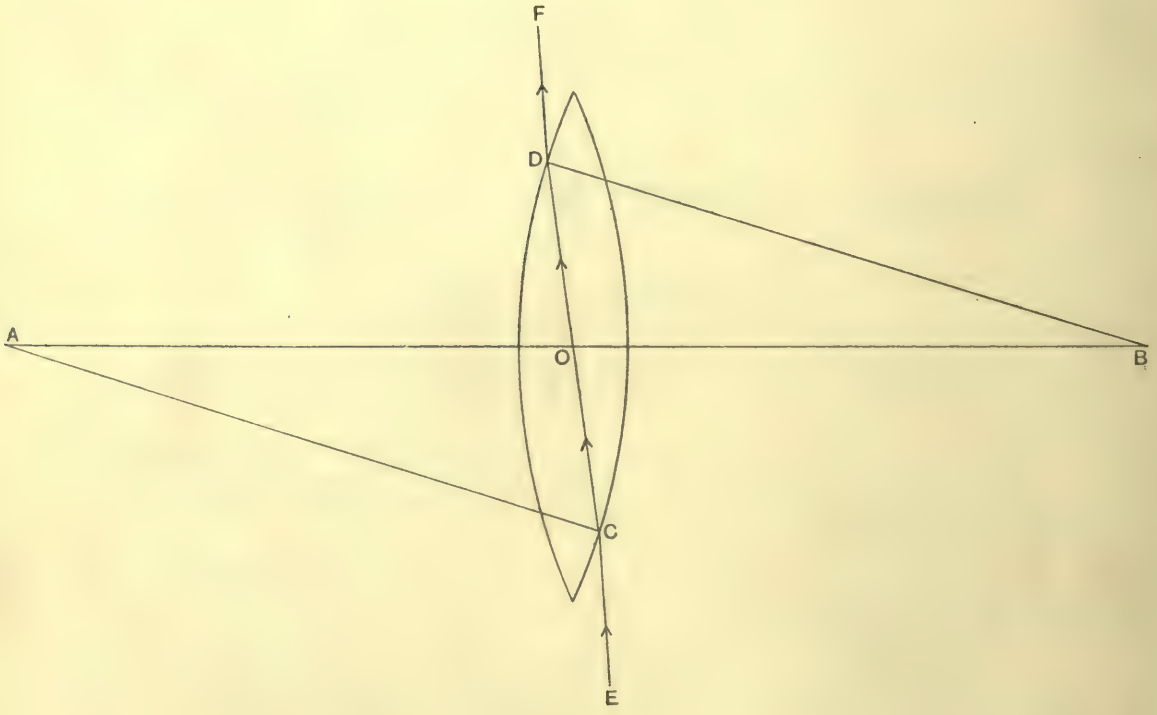
1. A convex lens may be looked upon as a collection of small prisms, whose angles get *larger* the further we go from the principal axis of the lens. This is shown by the diagram.

If a number of rays parallel to the principal axis meet such a series of prisms, the rays furthest from the principal axis will be *most* bent. The result is that the rays *converge* to a point on the principal axis, which is called the "principal focus" of the lens.

2. A concave lens may be looked upon in the same way as a collection of small prisms, whose angles get *larger* the further we go from the principal axis of the lens. This is shown by the diagram.

If a number of rays parallel to the principal axis meet such a series of prisms, the rays furthest from the principal axis will be *most* bent. The result is that the rays *appear to diverge* from a point on the principal axis, which is called the "principal focus" of the lens.


F
F


F
D
A
O
B
C
E

Def.

"The principal focus of a convex lens is a point on the principal axis *to* which rays, parallel to the principal axis, *converge* after refraction."

It is to the *left* of the lens.

Def.

"The principal focus of a concave lens is a point on the principal axis *from* which rays, parallel to the principal axis, *appear to diverge* after refraction."

It is to the *right* of the lens.

It is usual to denote the position of the principal focus by the letter F.

Def.

"The focal plane of a lens is a plane, perpendicular to the principal axis, which passes through the principal focus."

Optical Centre of a Lens

Let A and B be the centres of the two spherical surfaces of a lens.

Draw any two *parallel* radii AC, BD, to meet these surfaces and let the line CD represent a ray passing through the lens.

Now AC, BD, are normals to the surface at C and D, and, since they are parallel,

$$\angle ACD = \angle BDC.$$

$\therefore$ The ray CD makes equal angles with the normals at C and D.

$\therefore$ The incident and emergent rays, EC and DF, make equal angles with the normals at C and D, and are therefore *parallel.*

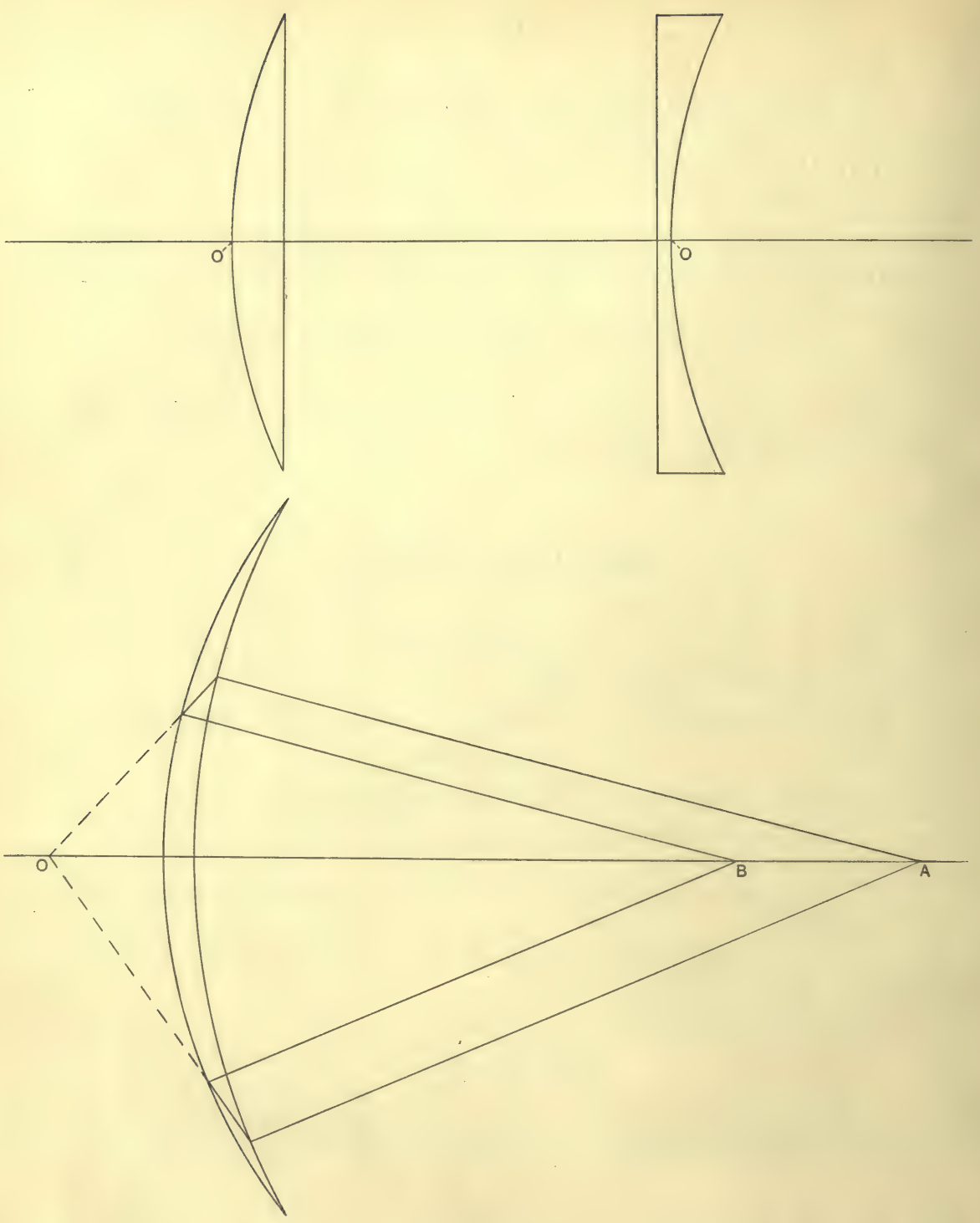
In other words the ray CD behaves as if it were passing through a parallel plate and is *undeviated.*

Let O be the point in which CD cuts the principal axis, and let R_1, R_2, be the radii of the spherical surfaces.

Then, in the similar triangles ACO, BDO,

$$\frac{AO}{BO} = \frac{AC}{BD} = \frac{R_1}{R_2}.$$

i.e. the point O divides the line AB in a definite ratio depending only upon the radii of the two spherical surfaces. The point O for a given lens is therefore fixed.


O
O
O
B
A

Two things follow from this:—

(1) Every ray whose direction on emergence is parallel to its original direction must pass through this point.

(2) Every ray which passes through this point has, on emergence, a direction which is parallel to its original direction.

The point O is called the "optical centre" of the lens.

We tested (1) by drawing two other parallel radii and found that the ray joining the two points passed through O. This is also clear geometrically from the similar triangles.

Again, if $R_1 = R_2$,

$$AO = BO.$$

i.e. O is the same as the *geometrical centre* of the lens.

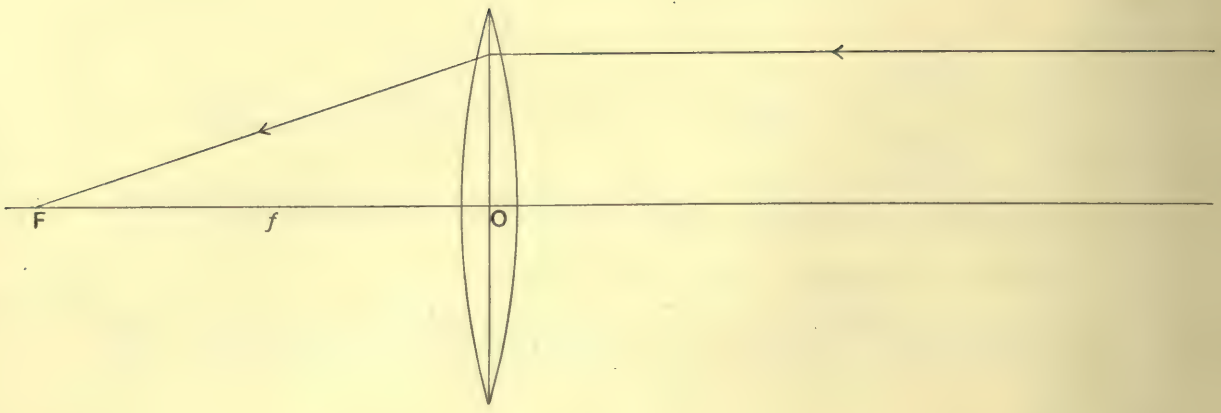
The position of the optical centre for various lenses is as follows:—

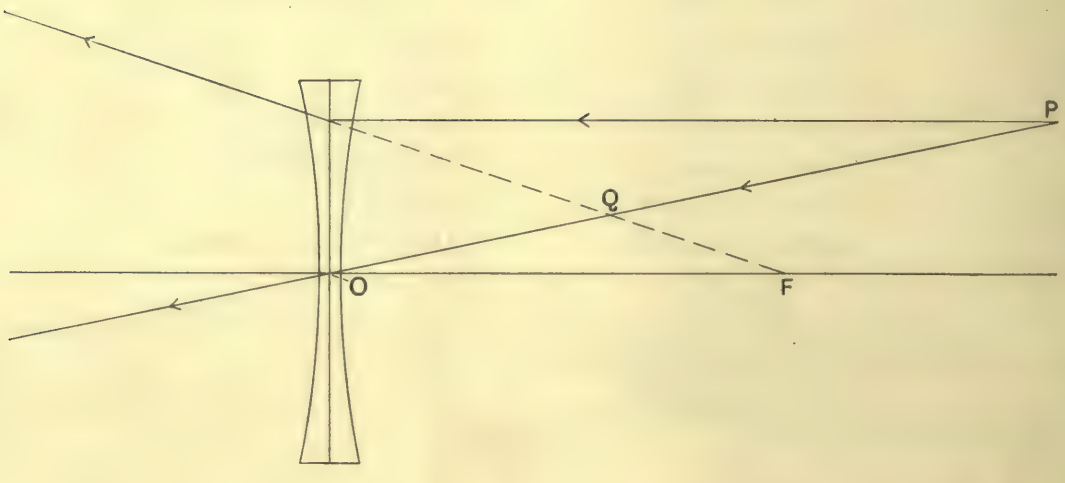
Double convex	... ...	Inside.
Double concave	... ...	Inside.
Plano-convex	... ...	On convex surface.
Plano-concave	... ...	On concave surface.
Convex meniscus	... ...	Outside.
Concave meniscus	... ...	Outside.

Def.

"The optical centre of a lens is a point on the principal axis such that any ray, which passes through it, suffers no deviation."

In the case of a meniscus the ray does not actually pass through the optical centre, but would do so, if produced.


F
f
O


P
Q
O
F

Def.

"The focal length of a lens is the distance between the optical centre and the principal focus of the lens."

Thin Lenses

In most of the lenses with which we have to deal the thickness of the lens is very small. The optical centre may therefore be taken to be the same as the geometrical centre.

Convention as to signs

Lines drawn to the *right* of a lens are taken as *positive*, whilst those drawn to the *left* are taken as *negative*.

Practical difficulty in Graphical Work

The lenses we deal with are supposed to be *very thin*, but in practice they have an appreciable thickness.

It is a good thing, therefore, to draw a line at right angles to the principal axis, cutting the lens into two portions, and to draw the rays up to this line in each case and *measure distances from it.*

To find by drawing the image of a point formed by refraction through a concave lens

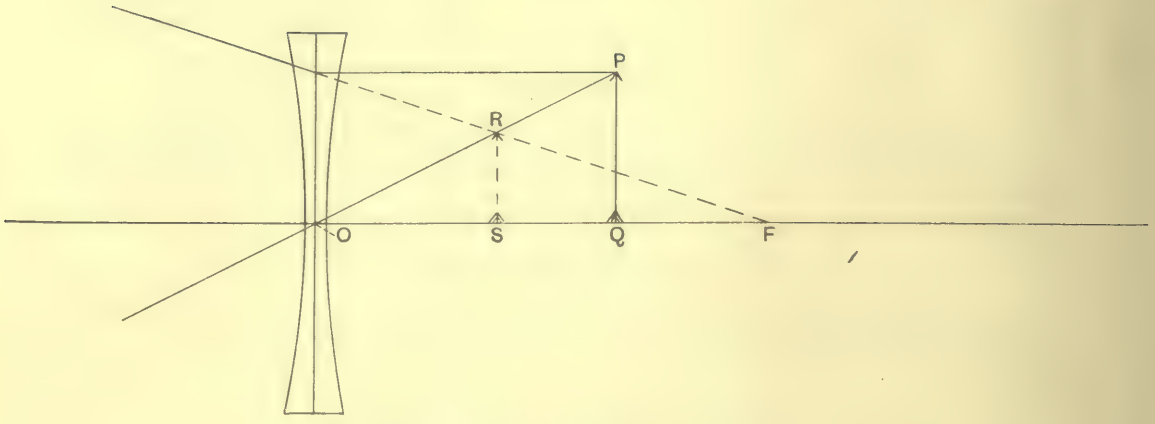
Experiments show that concave lenses produce images of points in front of them.

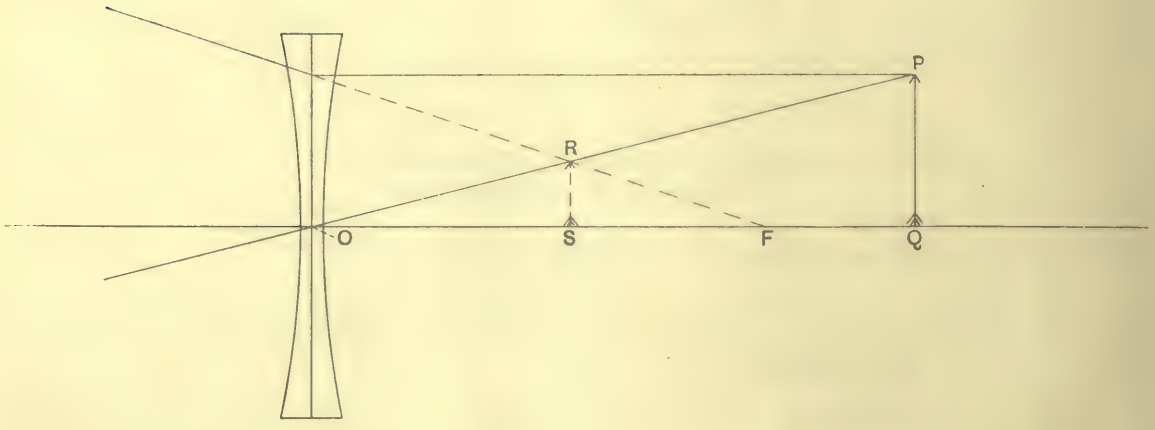
To determine the position of the image of a point it will be sufficient to trace the course of *two rays* and find where they intersect one another after refraction.

The two rays chosen are:—

1. A ray parallel to the principal axis, which, after refraction, appears to come from the principal focus.

2. A ray passing through the optical centre, which has its direction unchanged.


P
R
O
S
Q
F


P
R
O
S
F
Q

To find by drawing the image of an object formed by refraction through a concave lens

There are two possible positions for the object:—

1. Between the lens and *F*.

   The image is

   Virtual.

   Erect.

   Diminished.

   Between the lens and *F*.

2. Outside *F*.

   The image is

   Virtual.

   Erect.

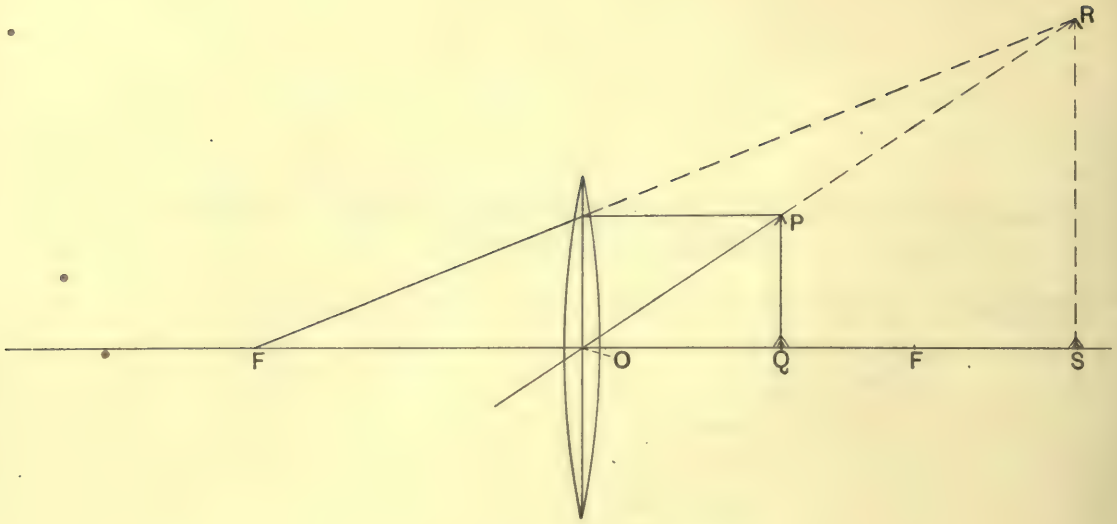
   Diminished.

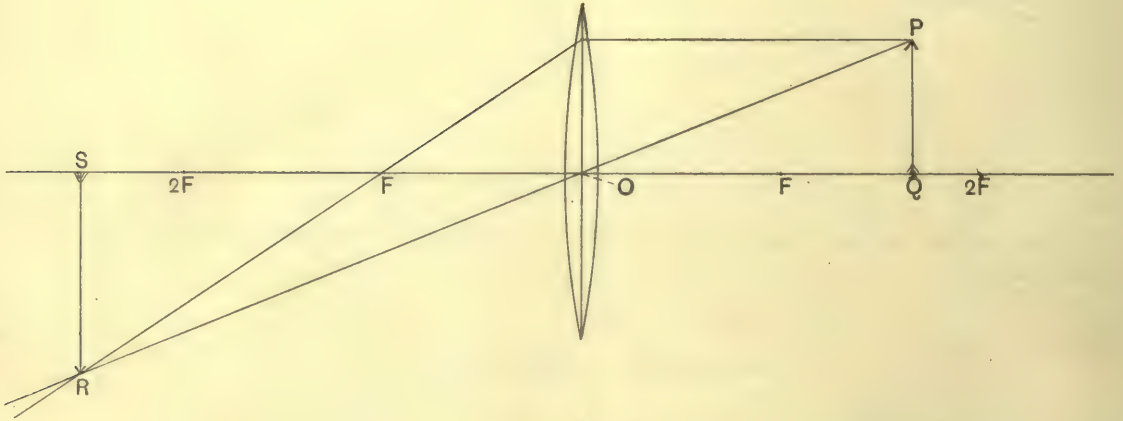
   Between the lens and *F*.

The position of the object therefore makes *no* difference, except as regards the size of the image.

Confirmation by Experiment

We took an object and put it in front of a concave lens in various positions.


R
P
F
O
Q
F
S


P
S
2F
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O
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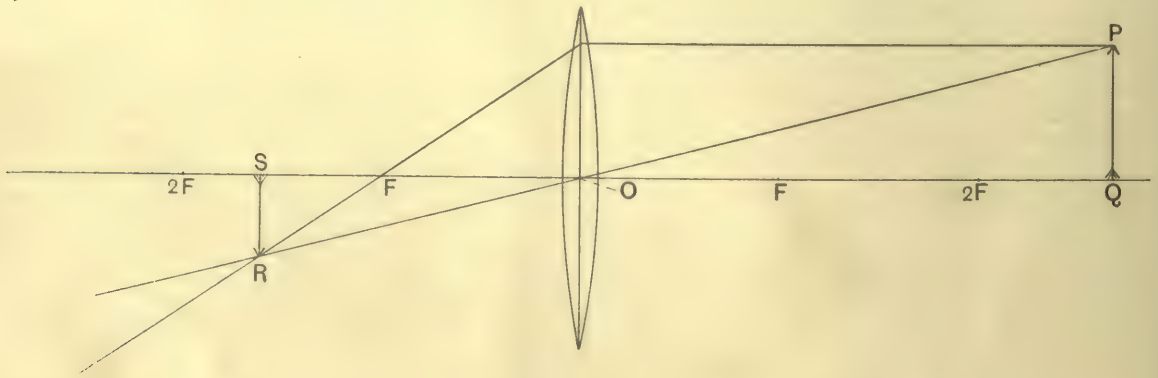
Q
2F
R


P
S
2F
F
O
F
2F
Q
R

To find by drawing the image of an object formed by refraction through a convex lens

The construction is almost the same as for a concave lens, the only difference being that the principal focus is on the left-hand side of the lens and the ray parallel to the principal axis passes through the principal focus.

There are three cases to consider :—

1. When the object is between the lens and F.

   The image is

   Virtual.

   Erect.

   Magnified.

   On the same side.

2. When the object is between F and $2F$.

   The image is

   Real.

   Inverted.

   Magnified.

   Outside $2F$.

3. When the object is outside $2F$.

   The image is

   Real.

   Inverted.

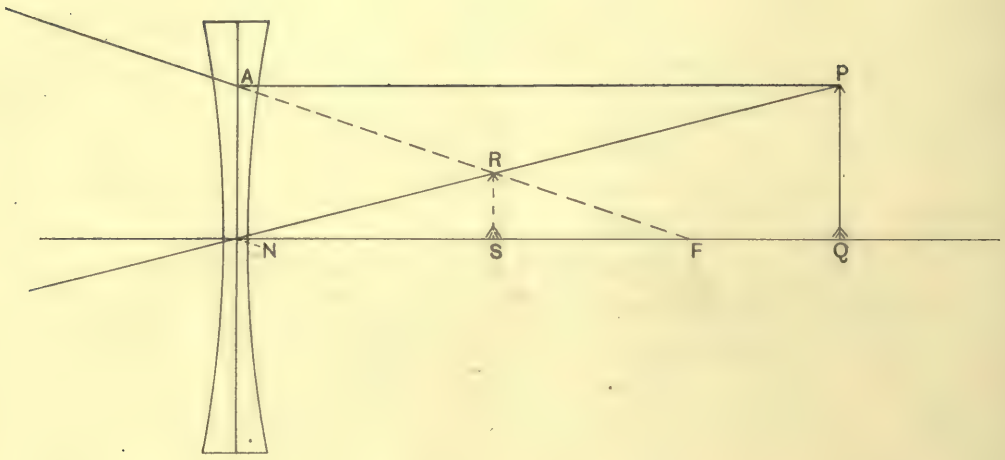
   Diminished.

   Between F and $2F$.

Confirmation by Experiment

We took a candle, put it in front of a convex lens in the three positions mentioned and tested the three cases.

We also noticed that, when the object is at $2F$, the image is at $2F$ on the other side and is of the same size.


A
P
R
N
S
F
Q

To trace the nature of the image with a convex lens as we pass from the lens outwards

Object.	Image.
1. Inside F.	Virtual, erect, magnified, same side.
2. At F.	At infinity.
3. Between F and $2F$.	Real, inverted, magnified, other side, outside $2F$.
4. At $2F$.	Real, inverted, same size, other side, at $2F$.
5. Outside $2F$.	Real, inverted, diminished, other side, between F and $2F$.

To obtain a formula for a concave lens

We drew the case for a concave lens.

Let u = distance of object from lens.

v = distance of image from lens.

f = focal length of lens.

In the similar triangles PNQ, RNS,

$$\frac{PQ}{RS} = \frac{NQ}{NS}. \qquad (1)$$

In the similar triangles AFN, RFS,

$$\frac{AN}{RS} = \frac{NF}{SF}.$$

But $AN = PQ$.

$$\therefore \quad \frac{PQ}{RS} = \frac{NF}{SF}. \qquad (2)$$

From (1) and (2),

$$\frac{NQ}{NS} = \frac{NF}{SF},$$

$$\frac{u}{v} = \frac{f}{f - v},$$

$$uf - uv = fv.$$

Dividing through by uvf,

$$\frac{1}{v} - \frac{1}{f} = \frac{1}{u}.$$

$$\therefore \quad \frac{1}{v} - \frac{1}{u} = \frac{1}{f}.$$

This is the *standard* formula for lenses and is applicable in *all* cases of lenses, if we remember to change the signs.

Rule for employing the formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

This formula can be used in *all* cases of lenses, if we remember that, when v and f are on the left of the lens, they must be given the *negative* sign.

Concave lens,

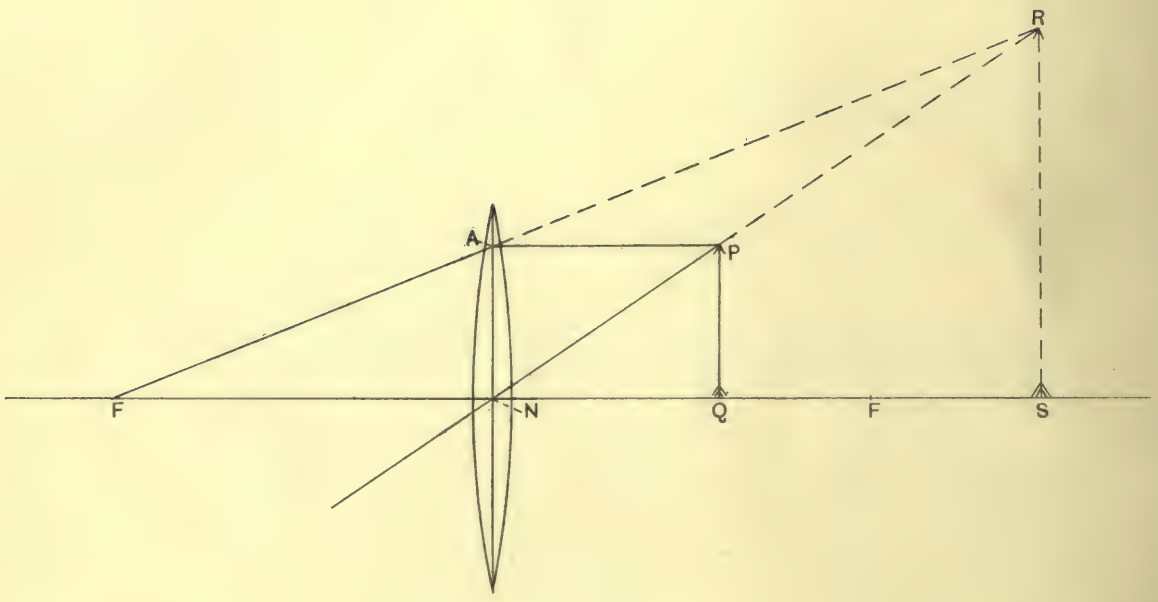
$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}.$$

Convex lens,

1. $\frac{1}{v} - \frac{1}{u} = -\frac{1}{f}.$

2. $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}.$

3. $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}.$


R
A
P
F
N
Q
F
S

To obtain a formula for a convex lens

We drew the case for a virtual, magnified image with a convex lens.

In the similar triangles PNQ, RNS,

$$\frac{PQ}{RS} = \frac{NQ}{NS}. \qquad (1)$$

In the similar triangles AFN, RFS,

$$\frac{AN}{RS} = \frac{FN}{FS}.$$

But $AN = PQ$.

$$\therefore \quad \frac{PQ}{RS} = \frac{FN}{FS}. \qquad (2)$$

From (1) and (2),

$$\frac{NQ}{NS} = \frac{FN}{FS},$$

$$\frac{u}{v} = \frac{f}{f+v},$$

$$uf + uv = fv.$$

Dividing through by uvf,

$$\frac{1}{v} + \frac{1}{f} = \frac{1}{u}.$$

$$\therefore \quad \frac{1}{v} - \frac{1}{u} = -\frac{1}{f}.$$

Note that this formula can be obtained from the *standard* formula

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

if we change the sign of f, which, for a convex lens, is negative, when we get

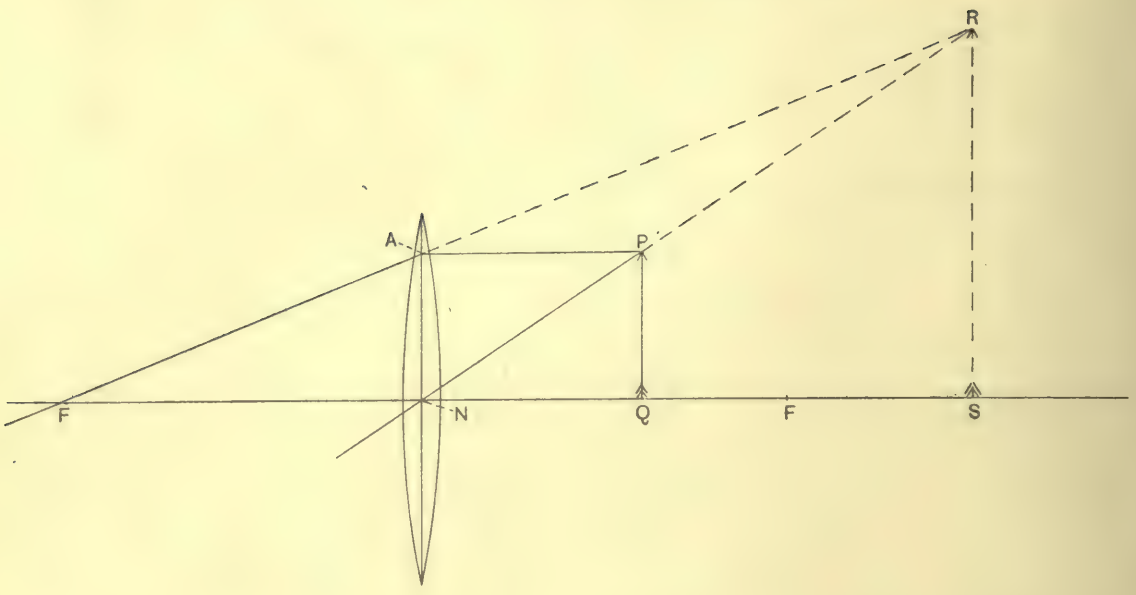
$$\frac{1}{v} - \frac{1}{u} = -\frac{1}{f}.$$

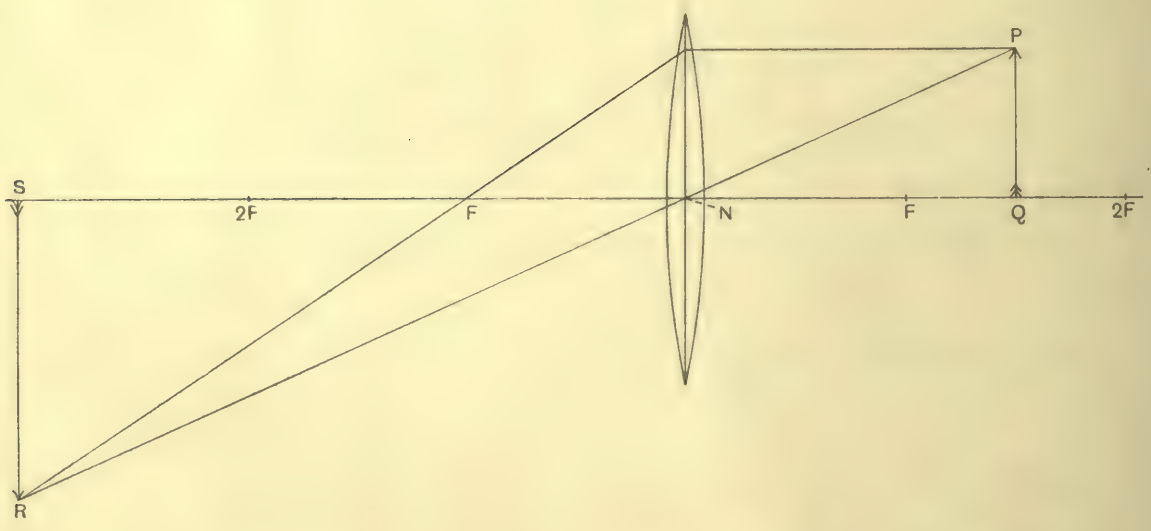
In the case where the image is real, v is also negative.

$$\therefore \quad -\frac{1}{v} - \frac{1}{u} = -\frac{1}{f},$$

$$\text{or} \quad \frac{1}{v} + \frac{1}{u} = \frac{1}{f},$$

i.e. they are all positive.


R
A
P
F
N
Q
F
S


P
S
2F
F
N
F
Q
2F
R

To find the magnification produced by a lens

We drew the case for a virtual image with a convex lens.

In the similar triangles RNS, PNQ,

$$\frac{RS}{PQ} = \frac{NS}{NQ} = \frac{v}{u}.$$

$$\text{Now Magnification} = \frac{\text{size of image}}{\text{size of object}}$$

$$= \frac{RS}{PQ}$$

$$= \frac{v}{u}.$$

To find the position for the object with a convex lens so that the real image obtained shall be twice the size of the object

Since the image is twice the size of the object,

$$\frac{v}{u} = 2.$$

$$\therefore\ v = 2u.$$

$$\text{Now}\quad \frac{1}{v} + \frac{1}{u} = \frac{1}{f}.$$

$$\therefore\ \frac{1}{2u} + \frac{1}{u} = \frac{1}{f},$$

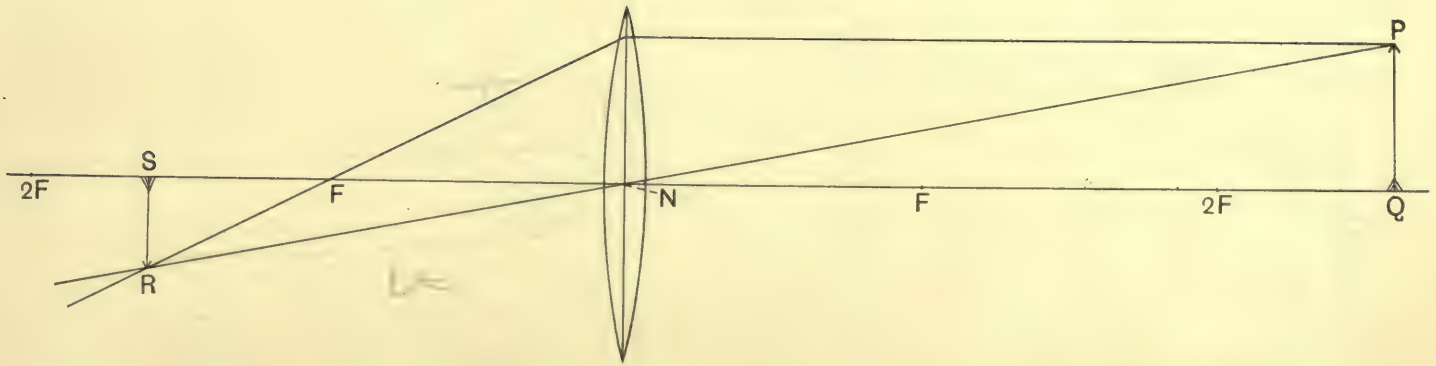
$$\frac{1+2}{2u} = \frac{1}{f},$$

$$\frac{3}{2u} = \frac{1}{f},$$

$$2u = 3f.$$

$$\therefore\ u = \frac{3}{2}f,$$

i.e. the object must be placed half-way between F and $2F$.


P
S
2F
F
N
F
2F
Q
R

Standard Type of Example

Find by drawing and calculation the position and size of the image of an object formed by refraction through a convex lens, given the following data :—

Focal length of lens = 5 cms.
Distance of object from lens = 13 cms.
Size of object = 2·5 cms.

The drawing must be done across the page.

1. By measurement.

Distance of image from lens = 8·10 cms.
Size of image = 1·55 cm.

2. By calculation.

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}.$$

$$\therefore \frac{1}{v}=\frac{1}{f}-\frac{1}{u}$$

$$=\frac{1}{5}-\frac{1}{13}$$

$$=\frac{13-5}{65}$$

$$=\frac{8}{65}.$$

$$\therefore v=\frac{65}{8}$$

$$=8{\cdot}13 \text{ cms.}$$

Distance of image from lens = 8·13 cms.

$$\text{Now } \frac{\text{Size of image}}{\text{Size of object}}=\frac{v}{u}.$$

$$\therefore \text{Size of image} = \text{size of object} \times \frac{v}{u}$$

$$=2{\cdot}5\times\frac{8{\cdot}13}{13}$$

$$=\frac{20{\cdot}33}{13}$$

$$=1{\cdot}56 \text{ cm.}$$

Size of image = 1·56 cm.

Combination of Two Lenses

Suppose we have two *thin* lenses of focal lengths f_1 and f_2. Considering the first lens of focal length f_1 and taking the standard formula

$$\frac{1}{v_1} - \frac{1}{u} = \frac{1}{f_1}. \qquad (1)$$

Considering the second lens of focal length f_2,

$$\frac{1}{v} - \frac{1}{u_2} = \frac{1}{f_2}.$$

Now the image formed by the first lens acts as the object for the second lens and, neglecting the thickness of the lenses, which are thin,

$$u_2 = v_1.$$

$$\therefore \quad \frac{1}{v} - \frac{1}{v_1} = \frac{1}{f_2}. \qquad (2)$$

Adding (1) and (2) we get

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2}.$$

Now in the case of rays from a distant object, which are parallel,

$$u = \text{infinity}.$$

$$\therefore \quad \frac{1}{u} = O.$$

$$\therefore \quad \frac{1}{v} = \frac{1}{f_1} + \frac{1}{f_2}.$$

Let F = focal length of combination.

Then, since parallel rays appear to diverge from the principal focus,

$$v = F.$$

$$\therefore \quad \frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}.$$

This is the *standard* formula for a combination of lenses.

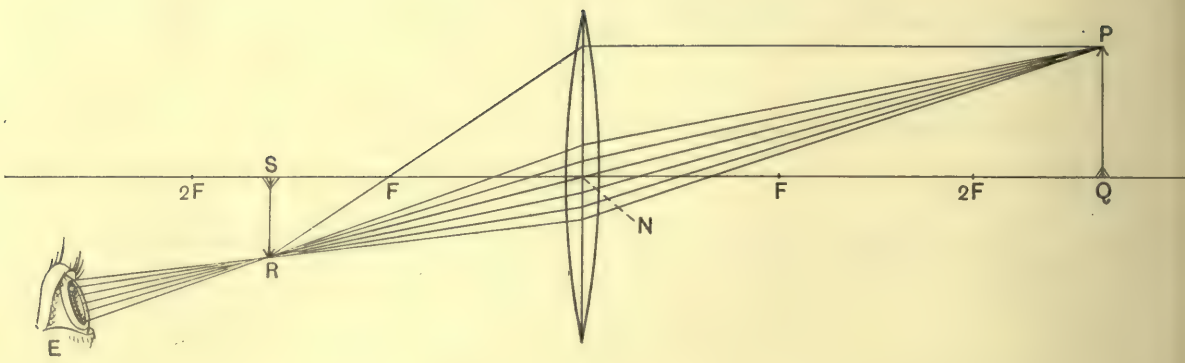
Let f_1 = focal length of concave lens.
f_2 = focal length of convex lens.
F = focal length of convex combination.

Then, putting in the correct signs, we get

$$-\frac{1}{F} = \frac{1}{f_1} - \frac{1}{f_2},$$

$$\text{or} \quad \frac{1}{f_1} = \frac{1}{f_2} - \frac{1}{F}.$$

It is usually f_1 that we require to find.


P
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2F
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2F
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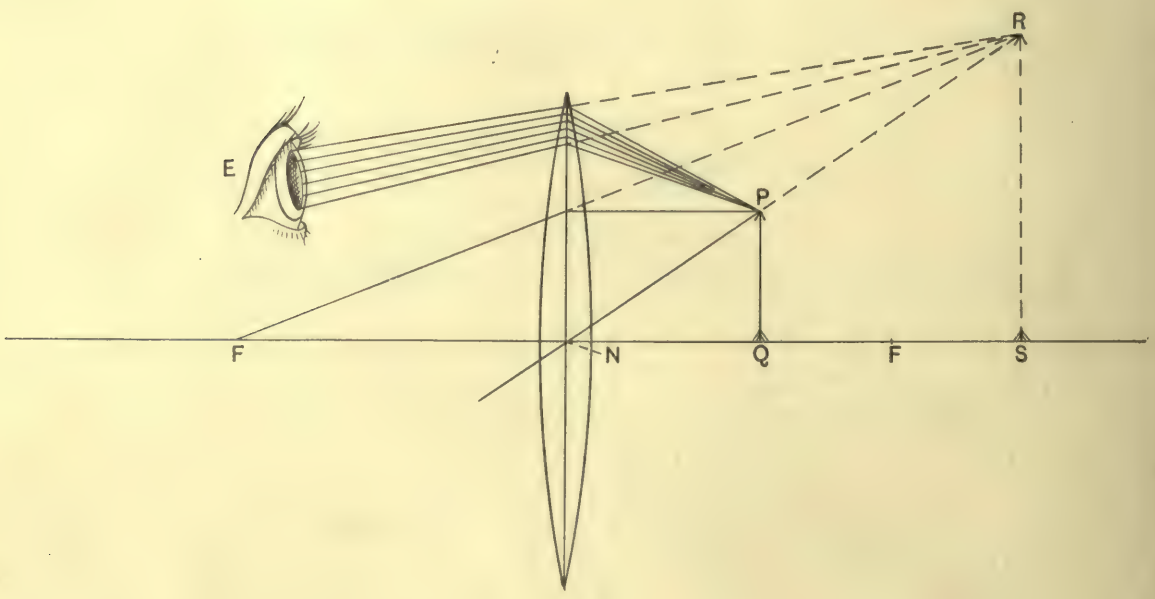

R
E
P
F
N
Q
F
S

This may also be obtained by taking the two formulae with the proper signs at once. Thus

$$\frac{1}{v_1} - \frac{1}{u} = \frac{1}{f_1}. \quad (1)$$

$$\frac{1}{v} + \frac{1}{v_1} = \frac{1}{f_2}. \quad (2)$$

Subtracting (2) from (1) we get

$$-\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} - \frac{1}{f_2},$$

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f_2} - \frac{1}{f_1}.$$

$$\therefore \quad \frac{1}{F} = \frac{1}{f_2} - \frac{1}{f_1},$$

$$\text{or} \quad \frac{1}{f_1} = \frac{1}{f_2} - \frac{1}{F}.$$

To draw the rays by which an eye sees the image of a point formed by refraction through a convex lens

I. Real Image

We drew a cone of rays, with the eye as base, to the image and produced the rays to the lens. We joined up the points, where the rays meet the lens, to the object.

II. Virtual Image

We drew a cone of rays, with the eye as base, to the image, dotting them where they do not exist. We joined up the points, where the rays meet the lens, to the object.

To find the focal length of a convex lens

I. Method of Parallel Rays

We arranged a lens in a holder so as to give a sharp image of a *distant* object on a screen held near it. We measured the distance of the screen from the lens.

Distance = 19·6 cms.

Since the rays from a distant object are parallel, they come to a focus at the principal focus or *in the focal plane of the lens.*

$\therefore$ Focal length of lens = 19·6 cms.

II. Method of Needles

We arranged a convex lens in a holder and placed on one side of it a knitting needle, so that it was outside the principal focus of the lens. We viewed the needle from the opposite side of the lens and placed a second needle to coincide with the real image of the first one seen through the lens.

We measured u and v, making a correction for the thickness of the lens by means of a pair of calipers.

$$u = 46{\cdot}6 \text{ cms.}$$
$$v = 61{\cdot}3 \text{ cms.}$$

Then $$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}.$$

$$\therefore \quad \frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$
$$= \frac{1}{61{\cdot}3} + \frac{1}{46{\cdot}6}$$
$$= \frac{46{\cdot}6 + 61{\cdot}3}{61{\cdot}3 \times 46{\cdot}6}$$
$$= \frac{107{\cdot}9}{2856{\cdot}58}.$$
$$\therefore \quad f = \frac{2856{\cdot}58}{107{\cdot}9}$$
$$= 26{\cdot}5 \text{ cms.}$$

Focal length of lens = 26·5 cms.

III. Optical Bench

1. We took the optical bench and arranged upon it an opaque screen, a lens in a holder, a piece of wire gauze in a holder and an electric light on a stand, so as to get a sharp image of the wire gauze on the screen.

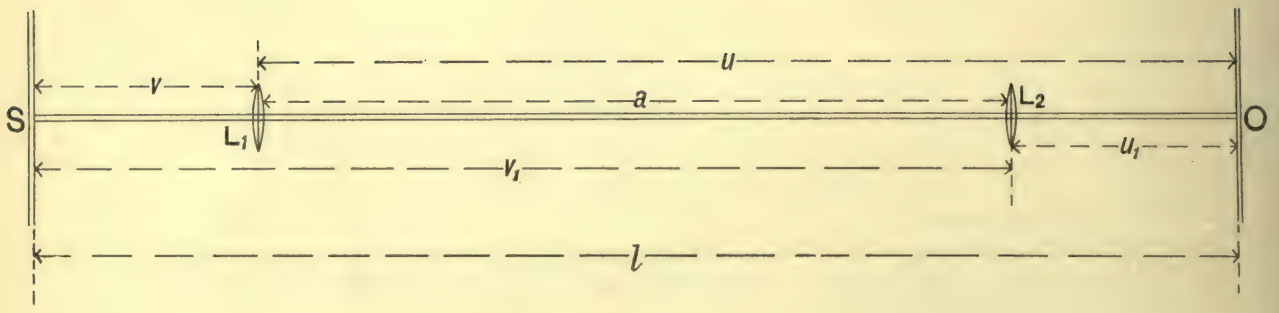
We measured u and v.

$$u = 24{\cdot}7 \text{ cms.}$$
$$v = 55{\cdot}3 \text{ cms.}$$

Then $$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}.$$

$$\therefore \quad \frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$
$$= \frac{1}{55{\cdot}3} + \frac{1}{24{\cdot}7}$$
$$= \frac{24{\cdot}7 + 55{\cdot}3}{55{\cdot}3 \times 24{\cdot}7}$$
$$= \frac{80{\cdot}0}{1365{\cdot}91}.$$
$$\therefore \quad f = \frac{1365{\cdot}91}{80{\cdot}0}$$
$$= 17{\cdot}1 \text{ cms.}$$

Focal length of lens = 17·1 cms.


S
O
L1
L2
v
u
a
v1
u1
l

2. In the preceding method there is always an error in measuring distances up to the lens. To overcome this error the following method is employed.

If the distance between the object and screen is more than $4f$, it can be shown that there are two positions for the lens in which an image is obtained :—

(1) When the object is between F and $2F$.

(2) When the object is outside $2F$.

Let O = object.

S = screen.

L_1 = 1st position of lens.

L_2 = 2nd position of lens.

We observed by experiment that

$$SL_1 = OL_2,$$

$$v = u_1.$$

This can also be proved mathematically in the following way :—

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f},$$

$$\frac{1}{v} + \frac{1}{a+u_1} = \frac{1}{f}. \qquad (1)$$

$$\frac{1}{v_1} + \frac{1}{u_1} = \frac{1}{f},$$

$$\frac{1}{a+v} + \frac{1}{u_1} = \frac{1}{f}. \qquad (2)$$

From (1) and (2),

$$\frac{1}{v} + \frac{1}{a+u_1} = \frac{1}{a+v} + \frac{1}{u_1},$$

$$\frac{1}{v} - \frac{1}{a+v} = \frac{1}{u_1} - \frac{1}{a+u_1},$$

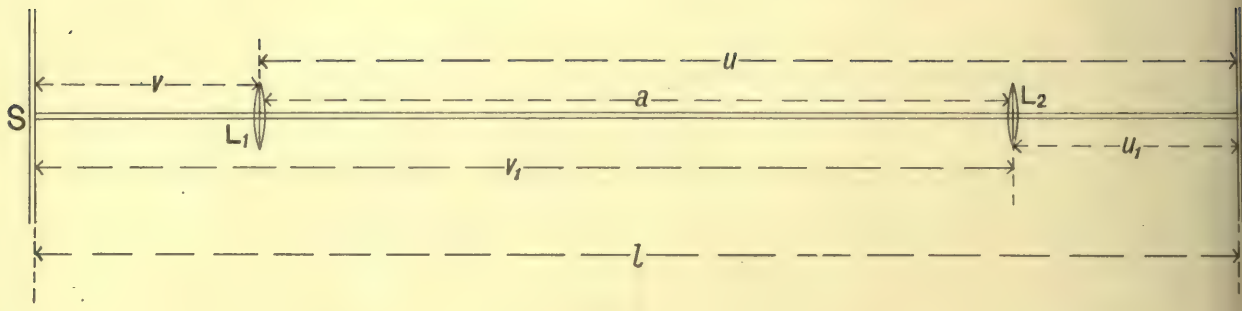
$$\frac{a+v-v}{v(a+v)} = \frac{a+u_1-u_1}{u_1(a+u_1)},$$

$$\frac{1}{v(a+v)} = \frac{1}{u_1(a+u_1)}.$$

$$\therefore \quad v(a+v) = u_1(a+u_1).$$

Now $v = u_1$ satisfies this equation ;

$$\therefore\ v = u_1.$$


S
v
u
a
L1
L2
v1
u1
l

Let l = distance between O and S,

a = distance between L_1 and L_2.

Then $u + v = l$,

$u - u_1 = a$.

But $u_1 = v$.

$\therefore \quad u - v = a$.

Thus we have

$$u + v = l,$$

$$u - v = a.$$

By adding,

$$2u = l + a.$$

$$\therefore \quad u = \frac{l + a}{2}.$$

By subtracting,

$$2v = l - a.$$

$$\therefore \quad v = \frac{l - a}{2}.$$

Then $$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

$$= \frac{1}{\frac{l - a}{2}} + \frac{1}{\frac{l + a}{2}}$$

$$= \frac{2}{l - a} + \frac{2}{l + a}$$

$$= \frac{2l + 2a + 2l - 2a}{l^2 - a^2}$$

$$= \frac{4l}{l^2 - a^2}.$$

$$\therefore \quad f = \frac{l^2 - a^2}{4l}.$$

In an experiment we obtained the following results:—

$$l = 80 \text{ cms.}$$
$$a = 55{\cdot}3 - 24{\cdot}7$$
$$= 30{\cdot}6 \text{ cms.}$$

Then $$f = \frac{l^2 - a^2}{4l}$$
$$= \frac{80^2 - (30{\cdot}6)^2}{320}$$
$$= \frac{6400 - 936{\cdot}36}{320}$$
$$= \frac{5463{\cdot}64}{320}$$
$$= \frac{546{\cdot}364}{32}$$
$$= 17{\cdot}1 \text{ cms.}$$

Focal length of lens = 17·1 cms.

To find the focal length of a concave lens

I. Method of Needles

We took a concave lens, which was cut in two across the diameter. We looked at a knitting needle through the lens and arranged a second needle so that it coincided with the image of the first seen through the lens.

We measured u and v.

$$u = 46{\cdot}0 \text{ cms.}$$
$$v = 12{\cdot}9 \text{ cms.}$$

Then $$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}.$$

$$\therefore \quad \frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$
$$= \frac{1}{12{\cdot}9} - \frac{1}{46{\cdot}0}$$
$$= \frac{46{\cdot}0 - 12{\cdot}9}{12{\cdot}9 \times 46{\cdot}0}$$
$$= \frac{33{\cdot}1}{593{\cdot}4}.$$
$$\therefore \quad f = \frac{593{\cdot}4}{33{\cdot}1}$$
$$= 17{\cdot}9 \text{ cms.}$$

Focal length of lens = 17·9 cms.

II. **Combination of a powerful convex lens with a concave lens to give a convex combination**

We have seen that, neglecting signs,

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}.$$

Now, if f_1 = focal length of concave lens,

f_2 = focal length of convex lens,

F = focal length of convex combination,

$$-\frac{1}{F}=\frac{1}{f_1}-\frac{1}{f_2},$$

$$\text{or } \frac{1}{f_1}=\frac{1}{f_2}-\frac{1}{F}.$$

We took a convex lens and found its focal length by means of the optical bench.

$u = 16{\cdot}0$ cms.

$v = 64{\cdot}0$ cms.

$$\text{Then } \frac{1}{f}=\frac{1}{v}+\frac{1}{u}$$

$$=\frac{1}{64{\cdot}0}+\frac{1}{16{\cdot}0}$$

$$=\frac{1+4}{64}$$

$$=\frac{5}{64}.$$

$$\therefore\ f=\frac{64}{5}$$

$$=12{\cdot}8 \text{ cms.}$$

Focal length of convex lens = 12·8 cms.

We then combined the convex lens with the concave lens, whose focal length we required, and obtained again a real image.

$$u = 34{\cdot}6 \text{ cms.}$$

$$v = 55{\cdot}4 \text{ cms.}$$

Then $$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

$$= \frac{1}{55{\cdot}4} + \frac{1}{34{\cdot}6}$$

$$= \frac{34{\cdot}6 + 55{\cdot}4}{55{\cdot}4 \times 34{\cdot}6}$$

$$= \frac{90{\cdot}0}{1916{\cdot}84}.$$

$$\therefore \quad f = \frac{1916{\cdot}84}{90{\cdot}0}$$

$$= 21{\cdot}3 \text{ cms.}$$

Focal length of convex combination = 21·3 cms.

Now we have seen that

$$\frac{1}{f_1} = \frac{1}{f_2} - \frac{1}{F},$$

where f_2 = focal length of convex lens

$= 12{\cdot}8$ cms.

F = focal length of convex combination

$= 21{\cdot}3$ cms.

$$\therefore \quad \frac{1}{f_1} = \frac{1}{f_2} - \frac{1}{F}$$

$$= \frac{1}{12{\cdot}8} - \frac{1}{21{\cdot}3}$$

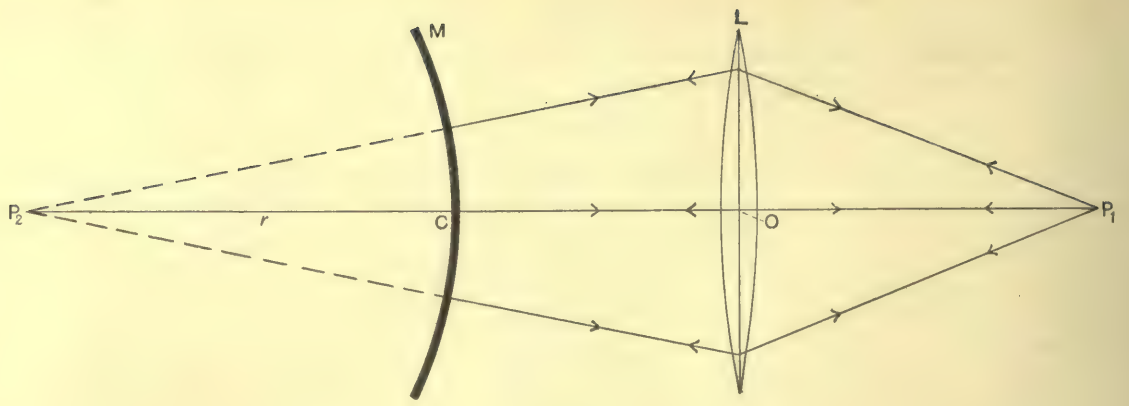
$$= \frac{21{\cdot}3 - 12{\cdot}8}{12{\cdot}8 \times 21{\cdot}3}$$

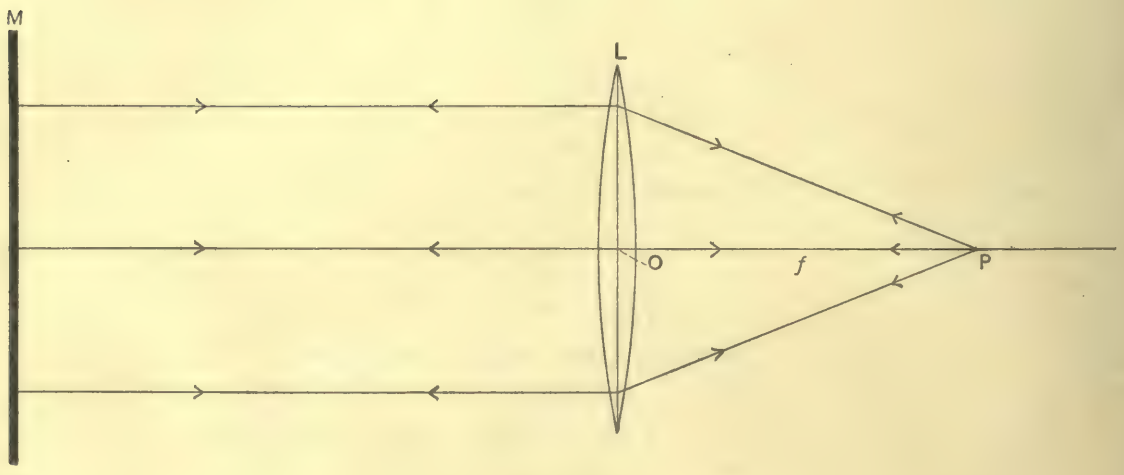
$$= \frac{8{\cdot}5}{272{\cdot}64}.$$

$$\therefore \quad f_1 = \frac{272{\cdot}64}{8{\cdot}5}$$

$$= 32{\cdot}1 \text{ cms.}$$

Focal length of concave lens = 32·1 cms.


M
L
P2
r
C
O
P1


M
L
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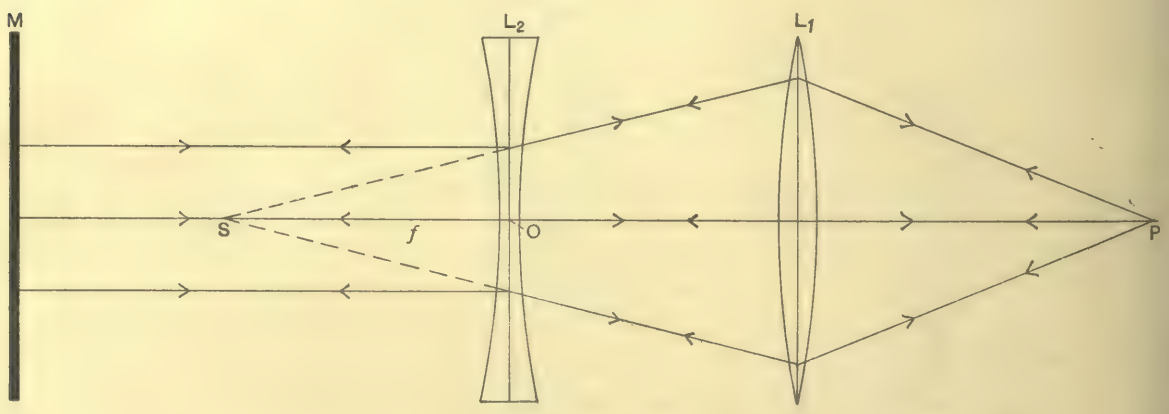
f
P


M
L2
L1
S
f
O
P

Special Methods for finding Focal Lengths

I. To find the focal length of a convex mirror by means of a convex lens

By this method it is *not* necessary to remove a strip of silvering.

We placed the convex mirror, M, about 20 cms. from a convex lens, L, of long focal length, and arranged a needle, P_1, in front of the lens in such a position that it coincided with its own image. When this is the case, the rays fall *normally* on the convex surface of the mirror and retrace their course. We measured the distance of the mirror from the lens, CO.

We then removed the mirror and obtained a coincidence of the image of P_1 with a second needle, P_2. We measured the distance of P_2 from the lens, P_2O.

The difference between these two measurements, $P_2O - CO$, gives the radius of curvature of the mirror, from which the focal length can be found.

II. To find the focal length of a convex lens by means of a plane mirror

Let M = plane mirror.
L = convex lens.
P = needle.

We arranged the needle to coincide with its own image. Then, since the rays which fall on M retrace their course, they must be parallel to the principal axis and therefore diverge from the principal focus.

We measured the distance OP, which is the focal length of the lens.

III. To find the focal length of a concave lens by means of a convex lens and a plane mirror

By this method it is *not* necessary to divide the lens into two parts.

We placed a pin at P and a convex lens, L_1, so as to give a real image of P at S. We arranged a plane mirror, M, and a concave lens, L_2, so that the image of P was coincident with itself.

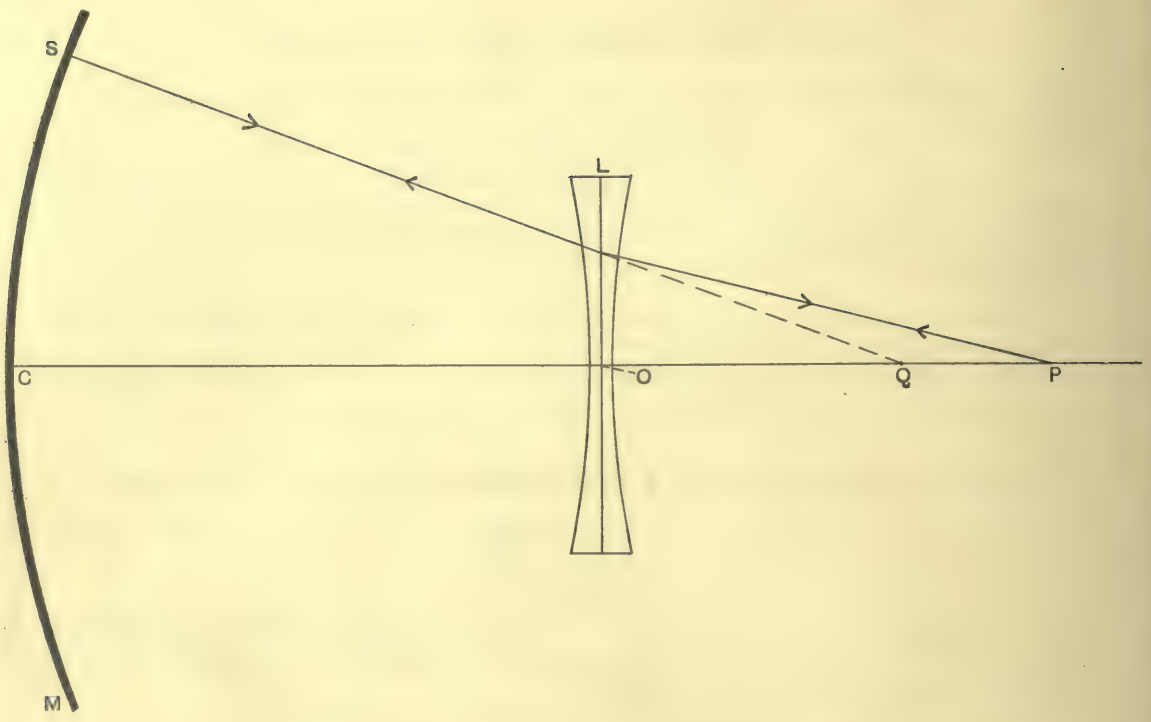
When this is the case, the rays, which were converging to S, go on parallel and are reflected back by the mirror along the same line. The distance OS is the focal length of the concave lens.

The rays, returning from the mirror parallel to the principal axis, *appear to diverge* from the principal focus.

We noted the position of O, removed the lens L_2 and found the position of S. We measured OS.

OS = focal length of concave lens.

In obtaining coincidence it is best to get a *blurred* image of the needle on interposing the concave lens, *i.e.* to get the rays going parallel. Then introduce the mirror and do the rest of the experiment by adjusting P and *not* L_2.


S
L
C
O
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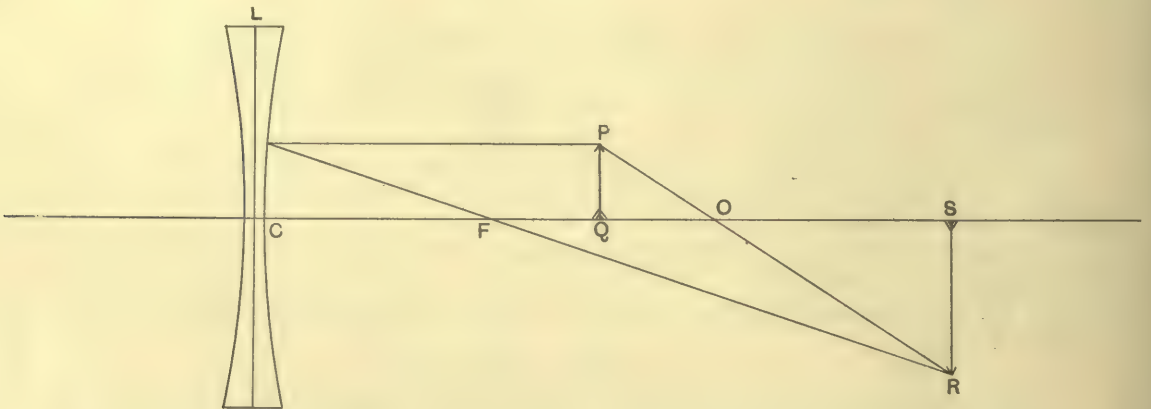
P
M


L
P
C
F
Q
O
S
R

IV. **To find the focal length of a concave lens by means of a concave mirror**

Let M = concave mirror.

L = concave lens.

P = needle.

f = focal length of lens.

We arranged the mirror so that an image of the needle, coincident with itself, was obtained. The ray from P, in passing through the lens, is refracted to S and appears to come from Q, which is the virtual image of P. If the ray QS strikes the mirror *normally*, it will go back along the same line.

It is clear that Q is the centre of curvature of the mirror and CQ the radius of curvature. We found the position of Q by removing the lens, and measured OP and OQ.

Then $OP = u$,

$OQ = v$,

and, for the lens, we know that

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f},$$

from which f can be found.

Special Methods for finding Radii of Curvature

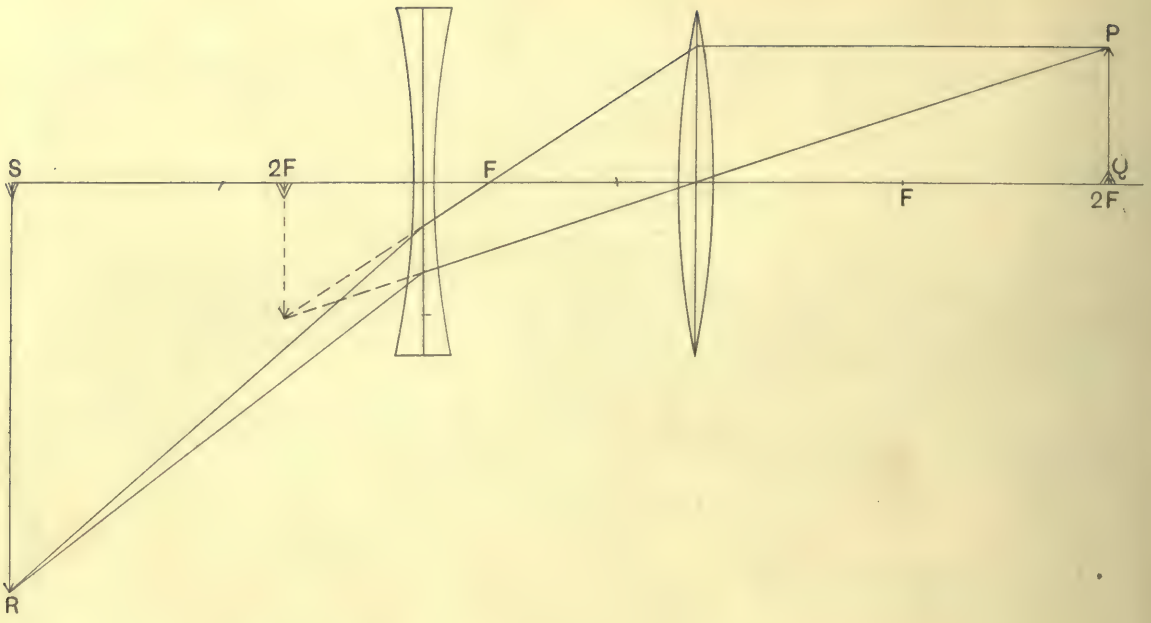
I. **To find the radius of curvature of the convex surface of a lens**

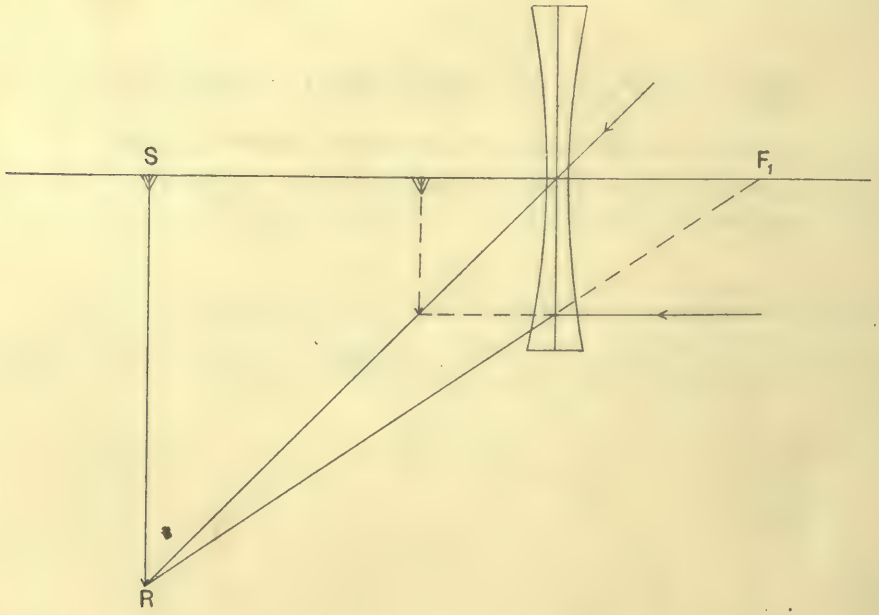
We may employ the method used for finding the focal length of a convex mirror. The convex surface of the lens behaves in the same way as the convex surface of the mirror.

II. **To find the radius of curvature of the concave surface of a lens**

The concave surface of the lens may be regarded as a concave mirror and its radius determined by obtaining the real image of an object in front of it and employing the formula

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}.$$


P
S
2F
F
F
2F
Q
R


S
F1
R

Combination of convex and concave lenses not in conjunction

A convex lens, having a focal length of 15 cms., is placed 20 cms. from a concave lens of the same focal length. An object is placed 30 cms. from the convex lens on the side *remote from* the other lens. Find the position of the image formed after refraction through both lenses.

1. By calculation.

Since the object is at $2F$, the first image is at $2F$, or at a distance of 30 cms., on the other side of the convex lens, *i.e.* 10 cms. to the left of the concave lens.

We then use the *standard* formula for a concave lens

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f},$$

but have to remember that u is to the *left* of the lens.

Then $$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}.$$

$$\therefore \quad \frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

$$= \frac{1}{15} - \frac{1}{10}$$

$$= \frac{2-3}{30}$$

$$= -\frac{1}{30}.$$

$$\therefore \quad v = -30 \text{ cms.}$$

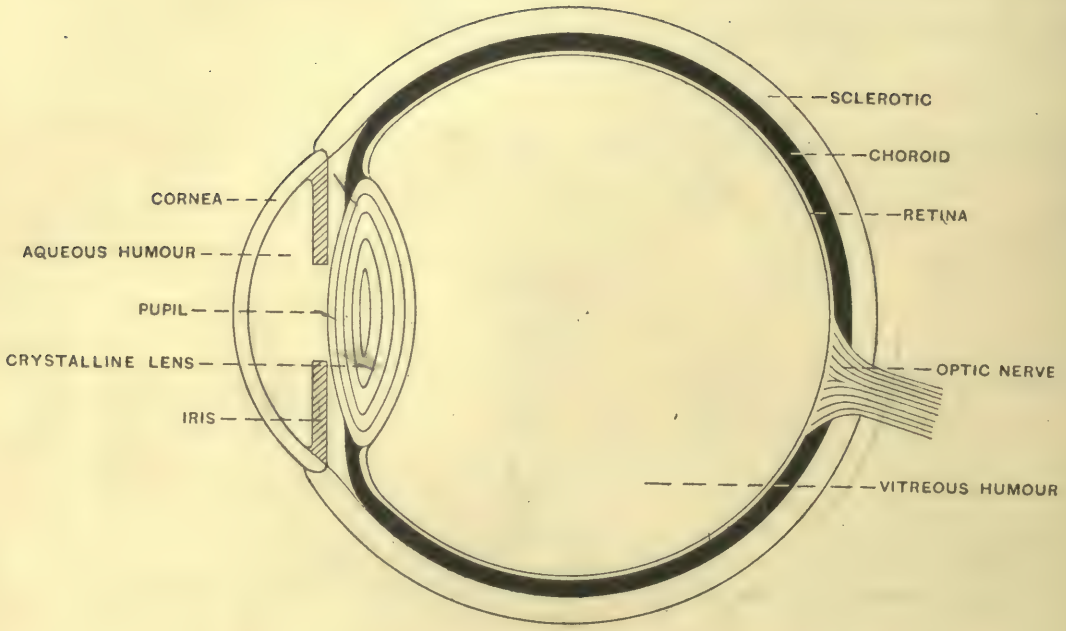
i.e. the image is 30 cms. to the left of the concave lens. It is a real image.

2. By drawing.

We choose two special rays, one a ray parallel to the principal axis and the other a ray passing through the optical centre.

To prove the formula $\frac{1}{f} = (\mu - 1)\left(\frac{1}{r} - \frac{1}{s}\right)$

(See advanced notes.)


SCLEROTIC
CHOROID
RETINA
CORNEA
AQUEOUS HUMOUR
PUPIL
CRYSTALLINE LENS
IRIS
OPTIC NERVE
VITREOUS HUMOUR

THE EYE

The human eye is a nearly spherical ball capable of turning in its socket. Its outermost coat is thick and horny, and it is opaque except in front. The opaque portion is the "sclerotic" or white of the eye. The transparent portion is called the "cornea" and has the shape of a very convex watch-glass. Behind the cornea is a diaphragm of annular form called the "iris." It is coloured and opaque, and the circular aperture in its centre is called the "pupil." By the action of the involuntary muscles of the iris this aperture is enlarged or contracted on exposure to darkness or light. It is the colour of the iris that gives a person eyes of a certain colour. Behind the pupil is the "crystalline lens," which has a greater convexity at the back than at the front. It is built up of layers or shells, increasing in density inwards, the outermost layer having nearly the same index of refraction as the medium in contact with it This is to prevent loss of light by reflection. The cavity between the cornea and the crystalline lens is filled with a watery liquid called the "aqueous humour." The much larger cavity behind the crystalline lens is filled with a transparent jelly, the "vitreous humour." This chamber is enclosed by the "choroid" coat, which is saturated with the "pigmentum nigrum." The choroid is lined with another membrane, the "retina," which is traversed by a system of nerve filaments, diverging from the "optic nerve." Light incident on the retina gives rise to the sensation of vision.

Summary of the various parts of the eye

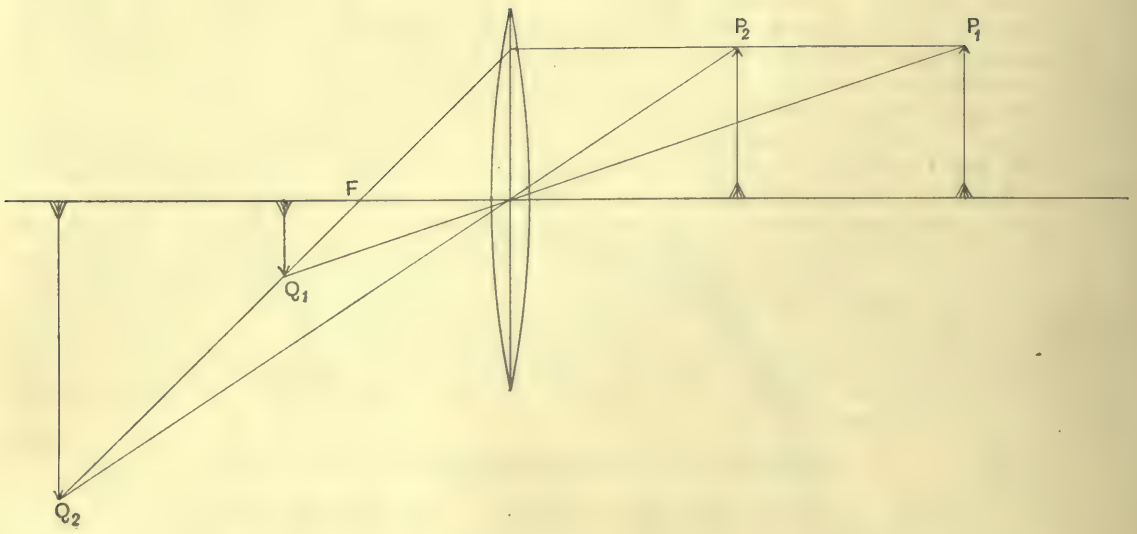
1. Sclerotic.—Thick and horny outer coat. This forms the white of the eye.
2. Cornea.—Transparent portion of the sclerotic, which is convex in shape.
3. Iris.—Diaphragm of annular shape, which is coloured. This gives the colour to the eye.
4. Pupil.—Circular aperture in the iris.
5. Crystalline lens.—Situated behind the pupil. It has a greater convexity at the back than at the front, and is built up of layers increasing in density inwards, the outermost layer having nearly the same index of refraction as the aqueous humour. This is to prevent loss of light by reflection.
6. Aqueous humour.—Watery liquid between the cornea and the crystalline lens.
7. Vitreous humour.—Transparent jelly behind the crystalline lens.
8. Choroid.—Inner coat of the eye, saturated with the "pigmentum nigrum."
9. Retina.—Membrane lining the choroid and traversed by a system of nerves.
10. Optic nerve.—Nerve by which the nerves of the retina are connected to the brain.

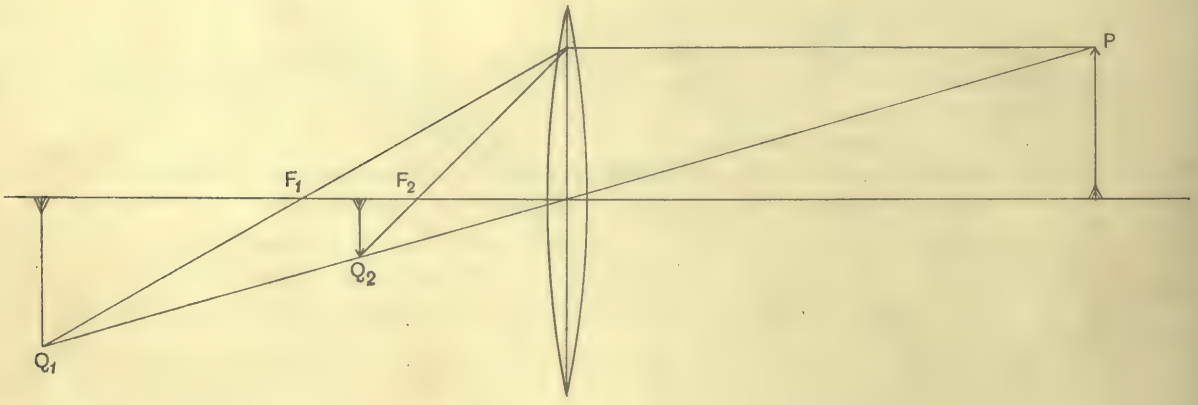
The straight line, which may be drawn through the centres of the cornea and crystalline lens, is called the "optic axis" of the eye.

The function of the iris is that of an adjustable stop, as in a photographic camera. It regulates the amount of light entering the eye.

In going into the light from a dark room the pupil contracts owing to the extension of the iris.

The function of the "pigmentum nigrum" of the choroid is probably to absorb the superfluous light. In the same way the insides of telescope tubes and photographic cameras are coated with dull black paint.


P2
P1
F
Q1
Q2


P
F1
F2
Q2
Q1

The Eye as an Optical Instrument

The cornea, aqueous humour, crystalline lens and vitreous humour constitute an optical system, which behaves like a convex lens of focal length 14·6 mm. When an object, placed at some distance from the eye, is viewed by it, you get a *real inverted image on the retina*. If the surfaces of the refracting media of the eye were perfectly *rigid*, distinct vision would only be possible in this one case, *i.e.* where the rays are parallel and come to a focus at the principal focus.

Accommodation

The crystalline lens is *not* perfectly rigid. By means of the "ciliary muscles," which connect the crystalline lens to the choroid, the lens can be made more convex. In this way the images of objects nearer to the eye can be brought on to the retina. This is called "accommodation." It is effected by an increase in the curvature of the *anterior* surface of the crystalline lens.

To show the need for Accommodation

We drew an object at two different distances from a convex lens and showed that the image went *further away* from the principal focus as the object came up towards the lens.

Q_1 is the image of P_1, and Q_2 of P_2.

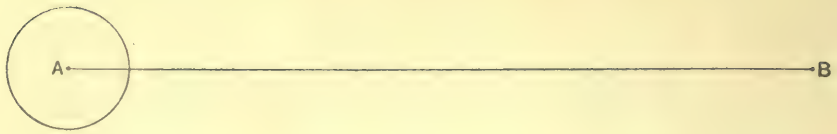
It is clear from the diagram that, if for one distance the image were on the retina, for the other distance it would be off the retina, provided that the lens were perfectly rigid.

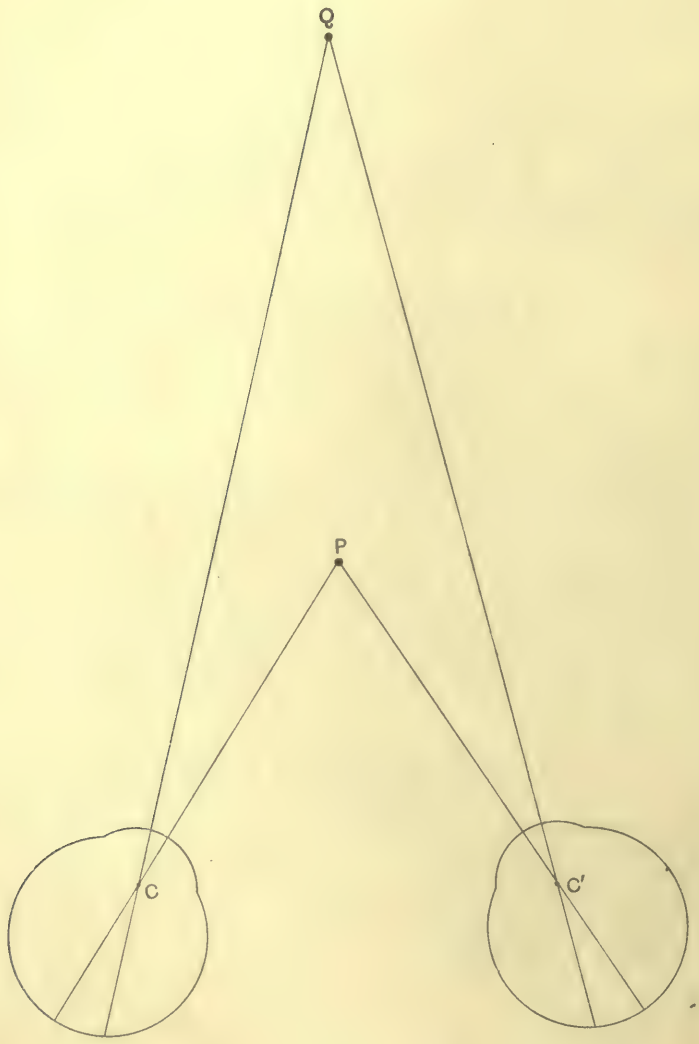
Persons with good sight can see objects clearly at all distances exceeding a certain limit. To examine the minute details of an object we hold it at a particular distance, about 8 ins. As we move it further away we experience more ease in looking at it and, as we bring it nearer the eye, we feel a certain amount of strain. At less than 6 ins. clear vision is impossible.

To show the result of Accommodation

We drew an object at some fixed distance from a convex lens and showed how the position of the image varied with the focal length of the lens, by varying the position of the principal focus, F_1 and F_2.

By shortening the focal length, *i.e.* making the lens *more convex*, the distance of the image from the lens is diminished.


A
B


Q
P
C
C'

The Blind Spot

We took two points, *A* and *B*, about 10 cms. apart. We put a sixpenny bit at *A*, closed the right eye and looked at *B*. The coin was invisible.

The explanation given is that there is a "blind spot" on the eye, where the optic nerve enters.

Binocular Vision

The reason that we see objects single with two eyes is that to each point on the retina of one eye there is a corresponding point on the retina of the other eye. An impression produced on one of these points is indistinguishable from a similar impression produced on the other, and, when they are both similarly impressed, the effect is simply more intense.

In order that we may see a point as single with two eyes, we must make its two images fall on corresponding points of the retina. If we hold up a finger between the eyes and the wall, and *look at the wall*, we see two *transparent* fingers projected on the wall. One is seen with either eye. It is important to note which eye sees which. To see the finger single we have to make the optic axes *converge more*, which requires some muscular effort.

The importance of this is that it is by this muscular effort that *we are able to judge the distance of objects from our eyes.* In fact, the principal advantage of having two eyes is in the *estimation of distance* and the *perception of relief.*

It is exceedingly difficult to estimate distance with only one eye.

e.g. (1) Threading a needle.

(2) Placing a finger on a spot.

Parallactic displacement, when we change our position, is also a great help in estimating distance.

Defects of Vision

I. Short-sight or Myopia

Distant objects are not seen distinctly. The eye-ball is *too long* for the lens or the lens is *too convex*. It is usually due to the first of these causes.

Parallel rays from a distant object are brought to a focus *in front of* the retina.

This is counteracted by a *concave* lens of suitable focal length. Its action may be looked at in two ways:—

1. As one of two lenses, it gives a smaller virtual image, nearer the eye, which is within the "distance of distinct vision."

2. As a combination of two lenses, it counteracts the too great convexity of the eye lens.

Distance of distinct vision

It is clear that there will be a distance *outside* which the eye *alone* cannot obtain distinct vision. The use of the concave lens is to bring the object *within* this distance.

Spectacles for Short-sight

They are *concave* and of such a focal length that, when an object is held about 10 ins. away, its virtual image is at the *nearest* distance of distinct vision for the person who is to use them. This latter distance must be ascertained by trial.

The reason for *nearest* distance of distinct vision is so as to get the image as *large* as possible.

Let p = nearest distance of distinct vision
$= 5$ ins. (say).
u = convenient distance for holding a book
$= 10$ ins. (say).

Then $v = p$
$= 5$ ins.

Now, for a concave lens,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}.$$

$$\therefore \quad \frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$= \frac{1}{5} - \frac{1}{10}$$

$$= \frac{2-1}{10}$$

$$= \frac{1}{10}.$$

$$\therefore \quad f = 10 \text{ ins.}$$

If his *greatest* distance of distinct vision exceed the focal length of the lens, he will be able, by means of the spectacles, to obtain distinct vision of objects at *all* distances from 10 ins. upwards. This is so since rays from a distant object, being parallel, appear to diverge from the principal focus, *i.e.* the image is at the principal focus. If this is *inside* his distance of distinct vision, it will be clear.

II. Long-sight or Hypermetropia

Near objects are not seen distinctly. The eye-ball is *too short* for the lens or the lens is *not sufficiently convex*. It is usually due to the first of these causes.

Rays from a near object are brought to a focus *behind* the retina.

This is counteracted by a *convex* lens of suitable focal length. Its action may be looked at in two ways:—

1. As one of two lenses, it gives a larger virtual image, further from the eye, which is at the nearest distance of distinct vision.

2. As a combination of two lenses, it increases the convexity of the eye lens.

Hypermetropia may be caused by lens changes, due to loss of accommodation, the lens becoming less convex. This frequently occurs with advancing years and is called "Presbyopia."

Spectacles for Long-sight

They are *convex* and of such a focal length that, when an object is held about 10 ins. away, its virtual image is at the *nearest* distance of distinct vision for the person who is to use them. This latter distance must be ascertained by trial.

Let p = nearest distance of distinct vision
$= 15$ ins. (say).
u = convenient distance for holding a book
$= 10$ ins. (say).

Then $v = p$
$= 15$ ins.

Now, for a *virtual* image with a convex lens,

$$\frac{1}{v} - \frac{1}{u} = -\frac{1}{f}.$$

$$\therefore \frac{1}{f} = \frac{1}{u} - \frac{1}{v}$$

$$= \frac{1}{10} - \frac{1}{15}$$

$$= \frac{3-2}{30}$$

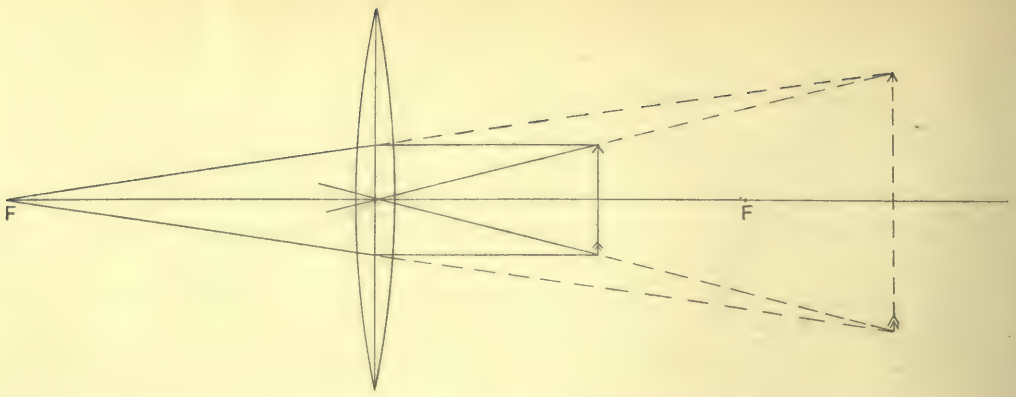
$$= \frac{1}{30}.$$

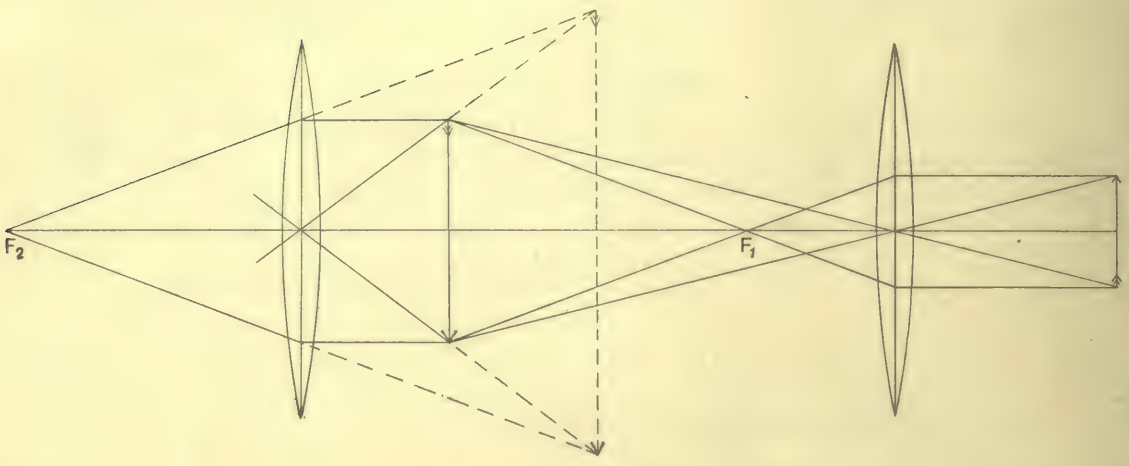
$$\therefore f = 30 \text{ ins.}$$

III. Astigmatism

The eye should be *symmetrical* about its axis, so that any section through the axis is equally curved and the focal lengths of all such sections are the same. Persons with such eyes see horizontal and vertical lines equally well at the same distance.

Sometimes there is *inequality in the curvature of the vertical and horizontal sections of the eye.* The *cornea* is usually the seat of this want of symmetry, being *more curved vertically than horizontally.* This irregularity is called "Astigmatism."


F
F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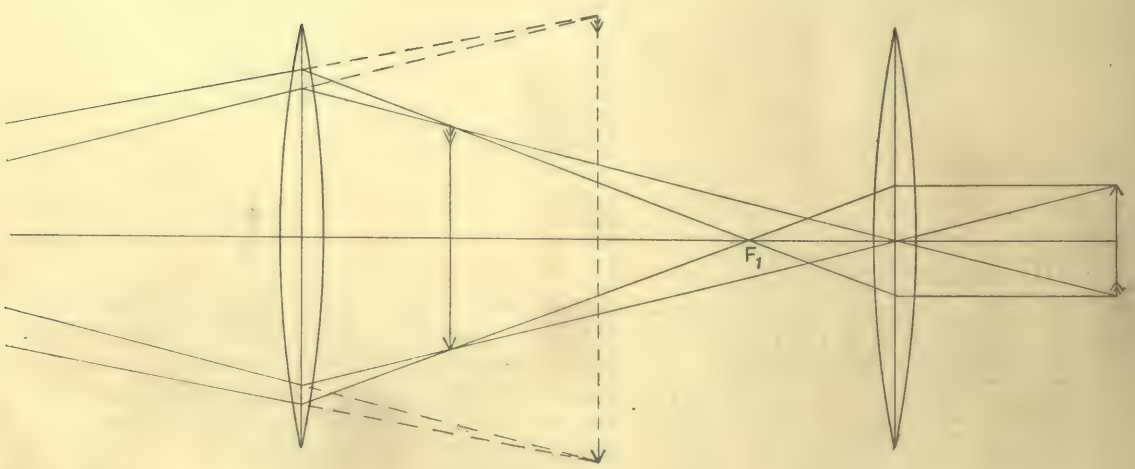

F1

OPTICAL INSTRUMENTS

Simple Microscope or Magnifying Glass

This consists of a convex lens placed at a distance somewhat less than its focal length from the object.

To make the magnification as large as possible, the focal length of the lens should be *small.* (See "Magnifying Power" in advanced notes.)

For the image to be properly seen, it must fall within the limits of distinct vision. To be seen under the largest possible visual angle, the eye must be close to the lens and the object must be as near as is compatible with distinct vision.

Compound Microscope

This consists of two lenses:—

1. Object lens or objective.

Convex lens, which forms a real and magnified image.

2. Ocular lens or eye-piece.

Convex lens, which magnifies the first image.

The object is placed *slightly* outside the principal focus of the objective.

The eye-piece is placed at a distance *a little less than* its focal length from the first image and this forms an enlarged virtual image. In practice the objective is composed of several lenses. They are achromatic combinations of flint and crown glass to prevent "chromatic aberration."

To make the magnification as large as possible, the focal length of the *objective* should be *small.* (See "Magnifying Power" in advanced notes.)

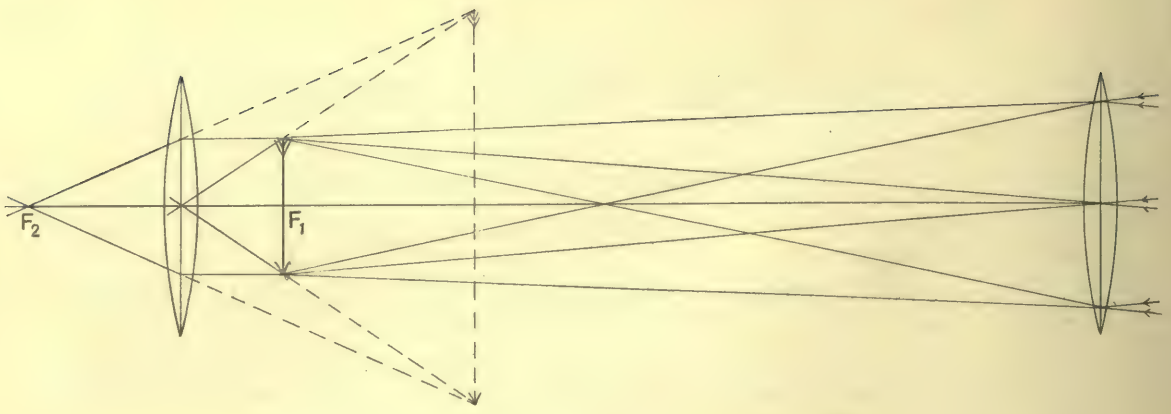
In practice the focal length of the *objective* is varied to give various magnifications.

It is important to note that we may draw the diagram for a compound microscope in two ways:—

(1) To show the images only.

(2) To show the rays which form the images going on to the eye.

The same thing is true of most of the diagrams in "Optical Instruments." The method we usually adopt is (1), which shows the images clearly and is, on the whole, simpler.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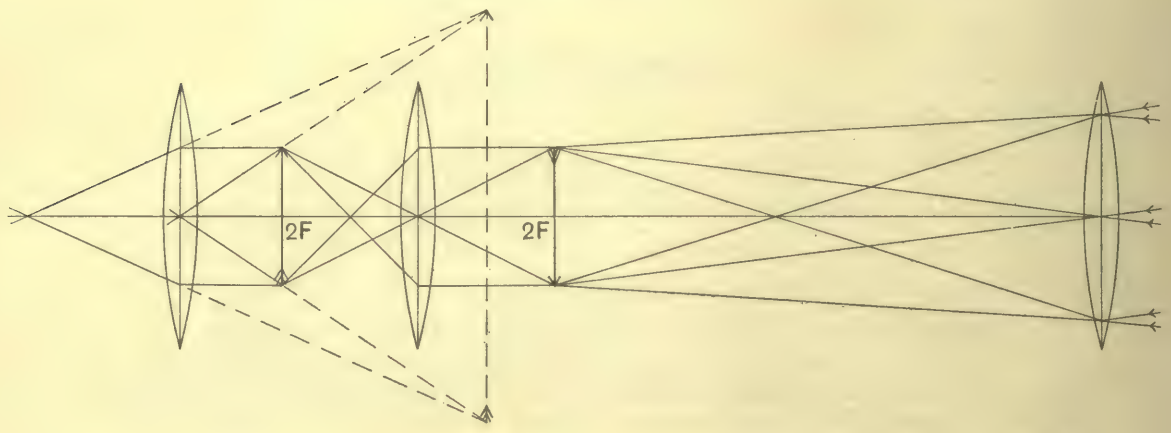
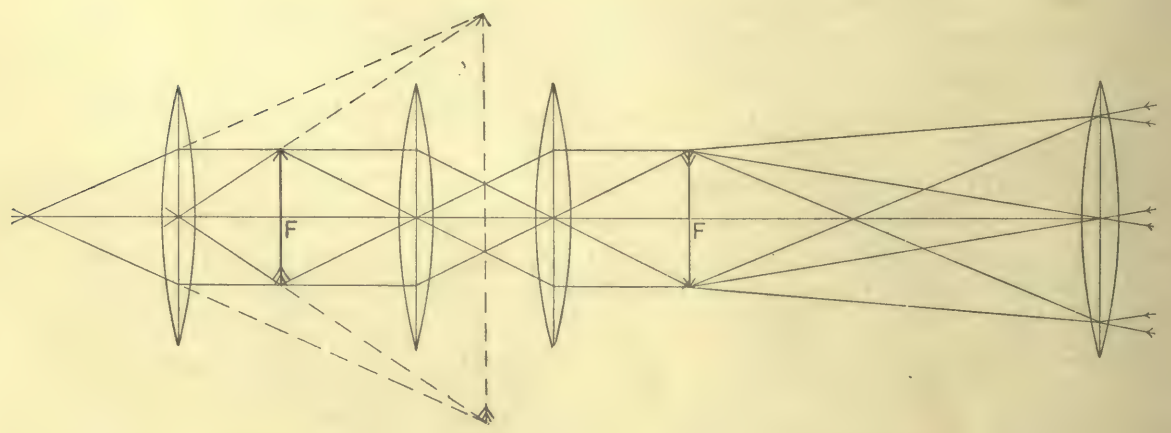

2F
2F


F
F

Astronomical Telescope

There are again two lenses:—

1. Objective.

This is sometimes 1 ft. or more in diameter and of much greater focal length than the eye-piece.

A real *inverted* image is formed *in the focal plane* of the lens. The image is obtained by drawing parallel rays from

(1) Upper extremity.

(2) Lower extremity.

2. Eye-piece.

This enlarges the real image produced by the objective.

The distance of the virtual image from the eye can be changed by pulling out or pushing in the eye-piece tube by means of a milled head. This is to get the image at the least distance of distinct vision. To make the magnification as large as possible, the focal length of the *objective* should be *large* and that of the *eye-piece small.* (See "Magnifying Power" in advanced notes.)

In practice the focal length of the objective is kept constant, as objectives are large and very expensive, and the focal length of the *eye-piece* is varied to give various magnifications.

Terrestrial Telescope

The Astronomical Telescope has this great disadvantage for terrestrial work, that the images are inverted.

There are two methods for re-inverting the images:—

1. A single convex lens is arranged so that the first image is at a distance $2f$ on one side. The second image is at a distance $2f$ on the other side.

2. A pair of convex lenses are arranged so that the first image is at a distance f from the first lens and that the lenses are a distance f apart. The second image is at a distance f from the second lens.

The objections to re-inversion are:—

(1) It always involves some loss of light and distinctness.

(2) It increases the length of the instrument.

e.g. In the first case we have 6 focal lengths and in the second 5.

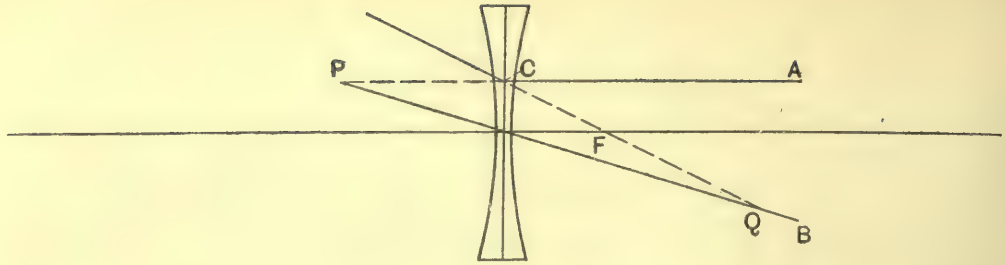
Best position for the eye

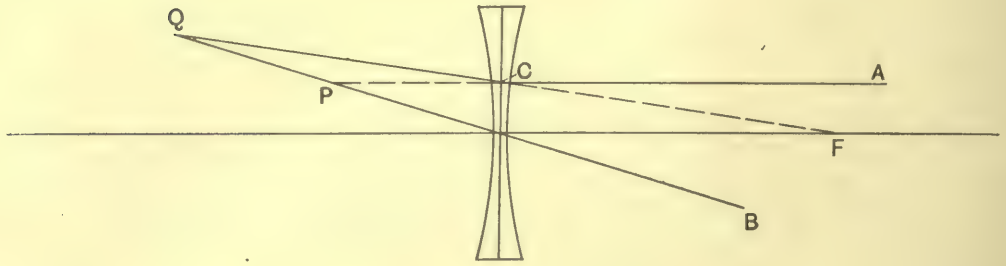
The eye-piece forms a real and inverted image of the objective, through which all the rays *transmitted by the telescope* must pass.

This can be tested by holding the telescope to a bright sky and getting the image on a piece of paper.

This image is the "bright spot" and is the proper place for the pupil of the eye. An eye at this point will see the *whole field of view* of the telescope.

To obtain the due position for the eye, a brass *diaphragm*, with a hole in its centre, is screwed into the eye end of the telescope, the proper place for the eye being close to this hole.


P
C
A
F
Q
B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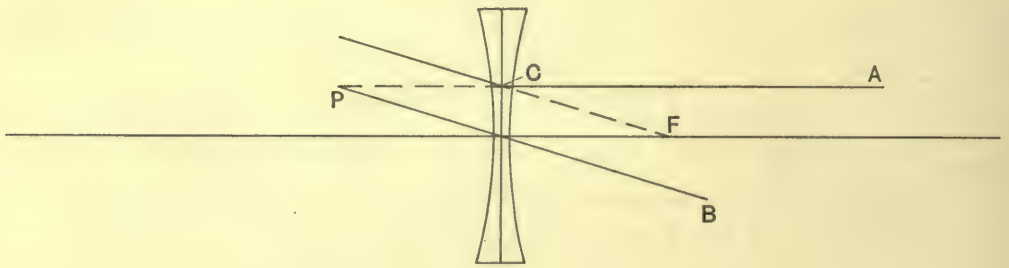
C
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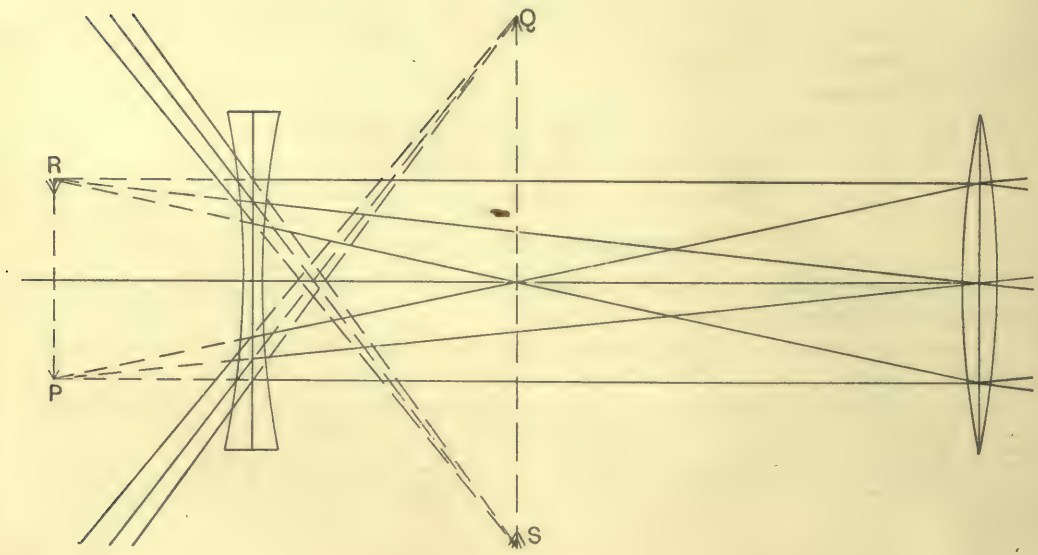

Q
R
P
S

Galileo's Telescope or Opera Glasses

The principle upon which this telescope depends is explained by the three diagrams.

Two rays AP, BP are converging towards P and a concave lens is interposed before they come to P.

Let f = focal length of lens.

PC = distance of P from lens.

1. If PC is greater than f, the rays *appear to diverge from* Q and we get a virtual magnified image.

2. If PC is less than f, the rays *converge to* Q and we get a real magnified image.

3. If $PC = f$, the rays *go parallel* and we get an image at infinity.

The case we employ in Galileo's telescope is (1), *i.e.* that in which PC is greater than f.

The concave lens is interposed at a distance *greater than* its own focal length, f, from the position of the image formed by a convex lens. But for the interposition of the convex lens we should get a real image at P. On interposing the concave lens the rays appear to come from Q.

Advantage of Galileo's Telescope

It is much shorter than the Terrestrial Telescope and yet gives erect images.

1. Terrestrial Telescope.

There are 5 or 6 focal lengths.

2. Galileo's Telescope.

There is *less* than 1 focal length.

Let f_1 = focal length of convex.

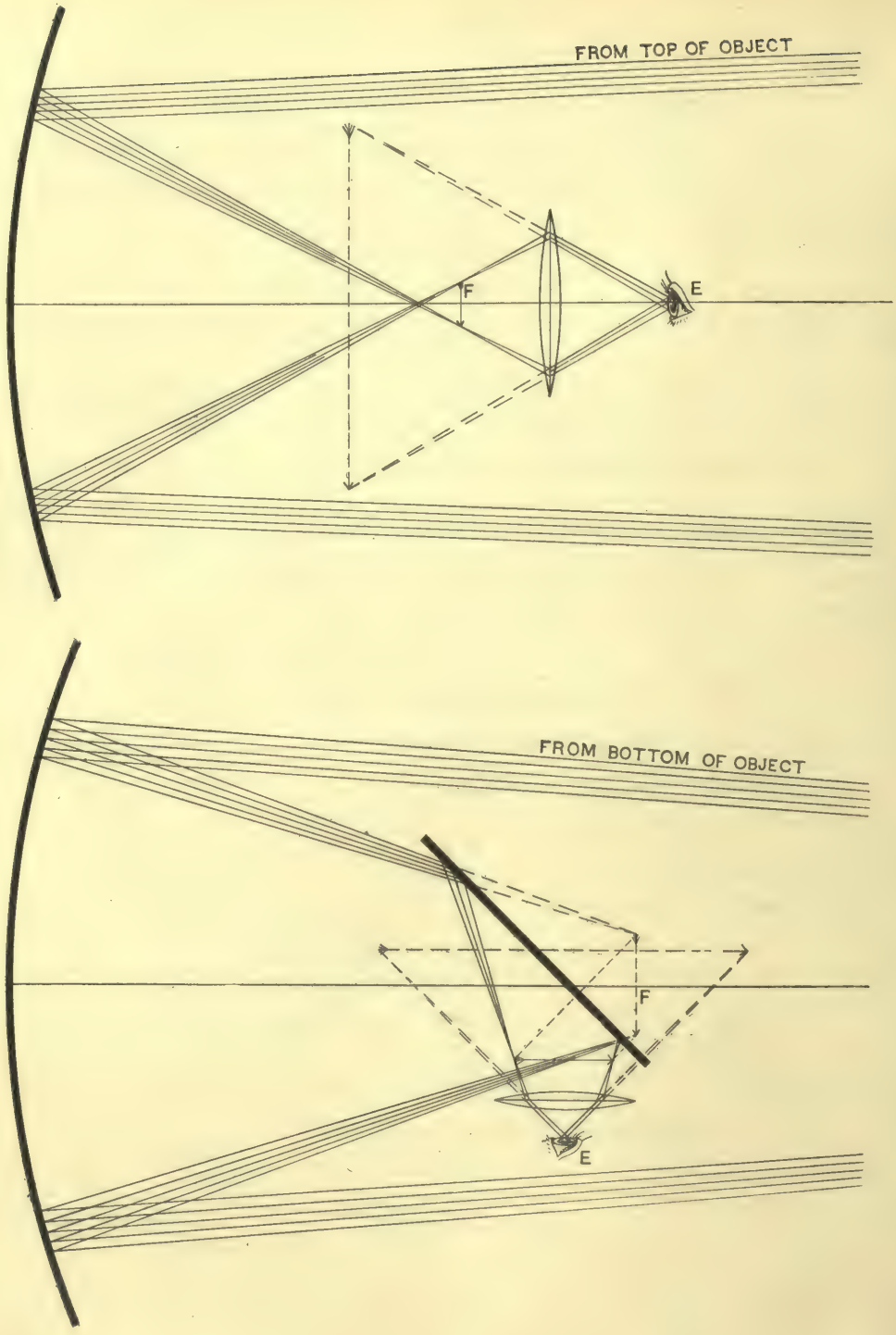
f_2 = focal length of concave.

x = distance $(PC - f_2)$.

Then, since $PC - f_2 = x$,

$$PC = f_2 + x.$$

$$\text{Length of instrument} = f_1 - PC$$
$$= f_1 - (f_2 + x)$$
$$= (f_1 - f_2) - x.$$


FROM TOP OF OBJECT
F
E
FROM BOTTOM OF OBJECT
F
E

Reflecting Telescopes

The objective is replaced by a concave mirror, called a "speculum," which gives a real inverted image.

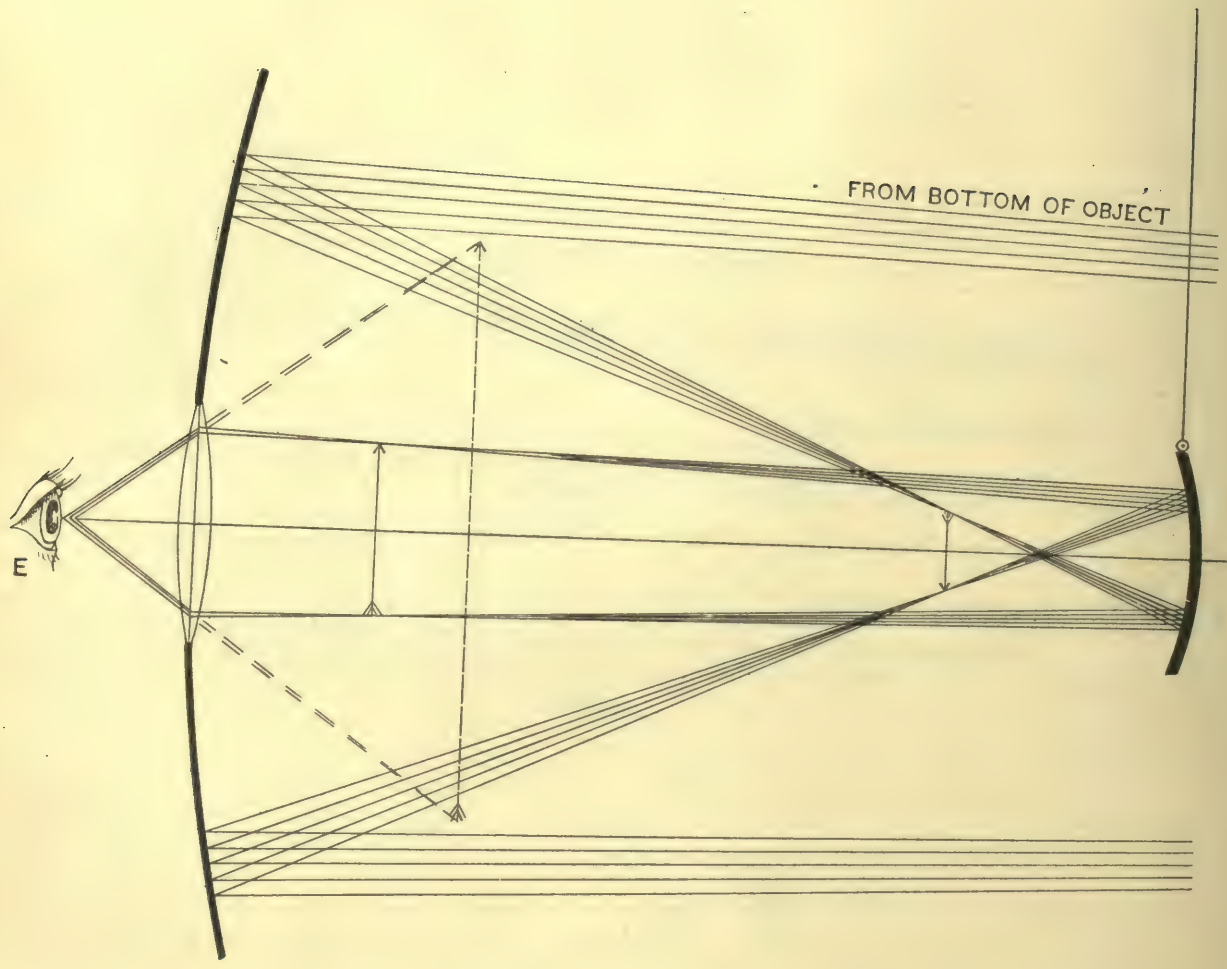
I. Herschelian Telescope

The image is viewed directly through an eye-piece, the back of the observer being towards the object and his face towards the speculum.

For this to be possible is required a *very large* speculum.

II. Newtonian Telescope

A *small* plane mirror is interposed *obliquely*, so that the real image is altered in position. This is viewed by an eye-piece, which gives the usual enlarged virtual image.


FROM BOTTOM OF OBJECT
E

III. Gregorian Telescope

There are two concave specula, one large and one small. The small one, which is suspended in the centre of the tube, receives the rays from the first speculum and forms a second real enlarged image of the first, which is erect. This image is magnified by an eye-piece, which is in a tube in the middle of the large speculum.

Advantages of this type :—

1. The image is erect.
2. The observer can look directly towards the object.

Advantage of Silvered Specula

Achromatic refracting telescopes give much the better results, but they are difficult to make of large size.

e.g. Diameter of specula = 6 ft.

Diameter of lenses = 3 ft.

Glass specula, covered with a thin coating of silver, are found to be the best, since glass is easier to work than speculum metal, which is an alloy of copper and tin.

Optical Lantern

The important parts are as follows:—

1. Arc Lamp or Limelight to produce a powerful light.

2. Condenser, consisting of a powerful double-convex lens, to condense or concentrate the rays.

3. Slide, to be strongly illuminated, placed a little way outside the focal length of the focussing lens. It is placed upside down.

4. Focussing Lens, to give a real *inverted* magnified image of the slide on a screen at a distance.

To focus the image we have to vary the position of the focussing lens.

To find the focal length of the focussing lens

We arranged a candle and a screen with the lens between, and got two images by varying the position of the lens.

We measured l and a.

$$l = 36 \text{ ins.}$$

$$a = 12{\cdot}5 \text{ ins.}$$

Then $$f = \frac{l^2 - a^2}{4l}$$

$$= \frac{(36)^2 - (12{\cdot}5)^2}{4 \times 36}$$

$$= \frac{1296 - 156{\cdot}25}{144}$$

$$= \frac{1139{\cdot}75}{144}$$

$$= 7{\cdot}91 \text{ ins.}$$

Focal length of lens $= 7{\cdot}91$ ins.

The lens was marked "8 ins. focus."

Photographic Camera

The important parts are as follows:—

1. A convex lens, at the front of the camera, to give a real *inverted* image of the object.

2. A sensitive plate, at the back of the camera, on to which the image is focussed.

3. A rack and pinion for varying the distance between the lens and the plate. For distant objects the plate is in the focal plane of the lens; for nearer objects the lens has to be "racked out," since the image is now outside f.

4. A stop or diaphragm, of adjustable size, by which the size of the aperture can be varied.

When the size of the aperture is reduced, the image of an object becomes sharper, owing to the cutting out of the marginal rays, for which the lens is not perfectly corrected.

A stop is measured by the number of times that its diameter is contained in the focal length of the lens.

Thus, if s = number of stop,

f = focal length of lens,

d = diameter of aperture,

$$s = \frac{f}{d}.$$

$$\therefore \quad d = \frac{f}{s}.$$

It is in this way that a stop is expressed, namely as a fraction of the focal length. Thus $f/8$ means that the diameter of the aperture is an eighth the focal length of the lens.

Again, since

$$d = \frac{f}{s},$$

$$ds = f.$$

Now, for a given lens, f is a constant.

$$\therefore \quad ds = \text{constant}.$$

$$\therefore \quad d \propto \frac{1}{s},$$

i.e. the diameter of the aperture is *inversely* proportional to the number of the stop.

But

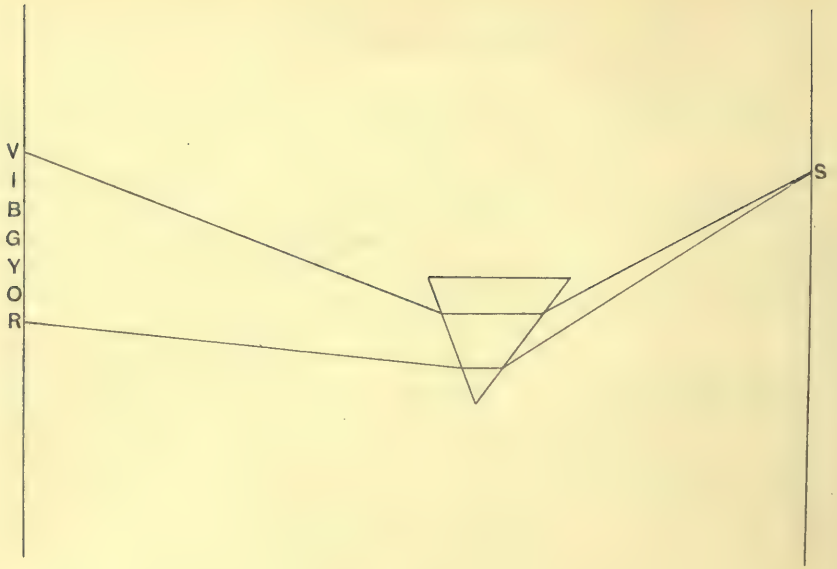
$$\text{area of aperture} \propto d^2.$$

$$\therefore \quad \text{area of aperture} \propto \frac{1}{s^2},$$

i.e. the area of the aperture is *inversely* proportional to the *square* of the number of the stop.

It is for this reason that the stops have the numbers 4, 5·6, 8, 11, 16, 22, 32, 45, the area of the aperture corresponding to each stop being approximately *half* that of the preceding one. This can readily be tested.

Now, since the quantity of light entering the camera is *directly* proportional to the area of the aperture, it follows that, if the stop is changed from 4 to 5·6, or from 11 to 16, or from 32 to 45, it will be necessary to *double* the exposure, since the area of the aperture in each case will have been halved.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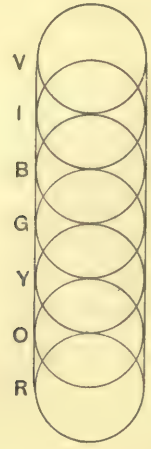
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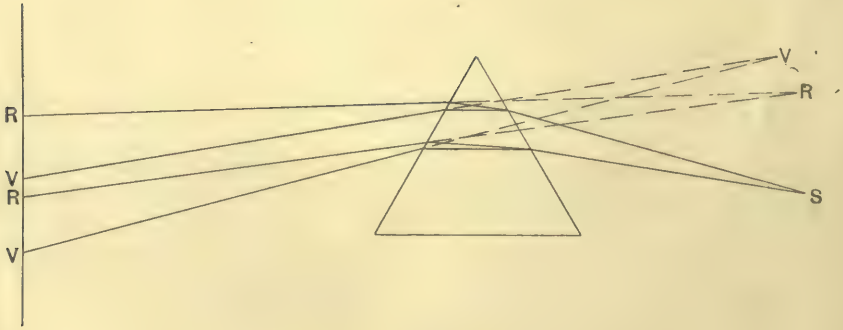
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THE SPECTRUM

There are two kinds of Spectra :—

1. Impure.
2. Pure.

Impure Spectrum

Newton's Experiment

A beam of sunlight is admitted through a small opening into a dark room. If allowed to fall normally on a white screen, it produces a *round* white spot, which is an image of the sun. If a prism is placed in its path edge *downwards* two effects are obtained :—

1. The beam is deflected upwards, *i.e.* away from the edge.
2. The beam is split up into its component colours forming the solar spectrum.

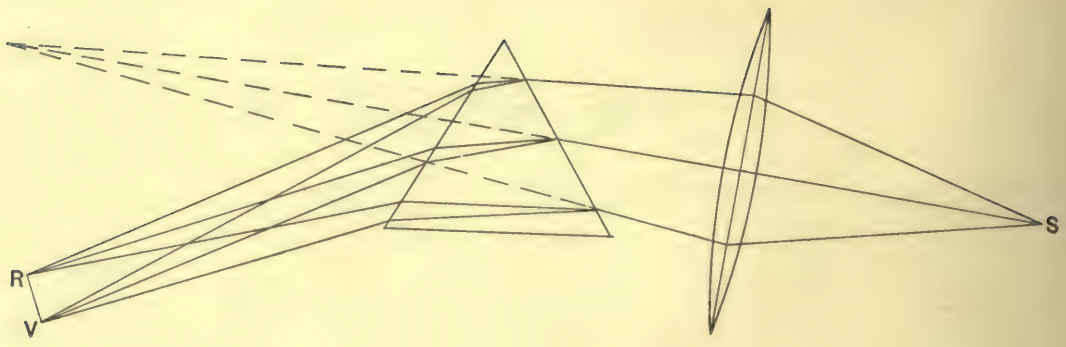
Violet. Most bent.
Indigo.
Blue.
Green.
Yellow.
Orange.
Red. Least bent.

The image will be a rectangle rounded off at the ends and having the same width as the original white image. It is formed by the *overlapping* of a number of round images of different colours, as shown in the second diagram.

If a further prism is employed, the light passing through a small opening in a screen, the coloured rays will be still further separated. The spectrum will thus become *less impure.*

This is a *real* spectrum.

The way in which the overlapping takes place is shown by the third diagram, in which the prism is placed with its edge *upwards.* In drawing this it is better to draw the virtual image first and so get the direction of the rays.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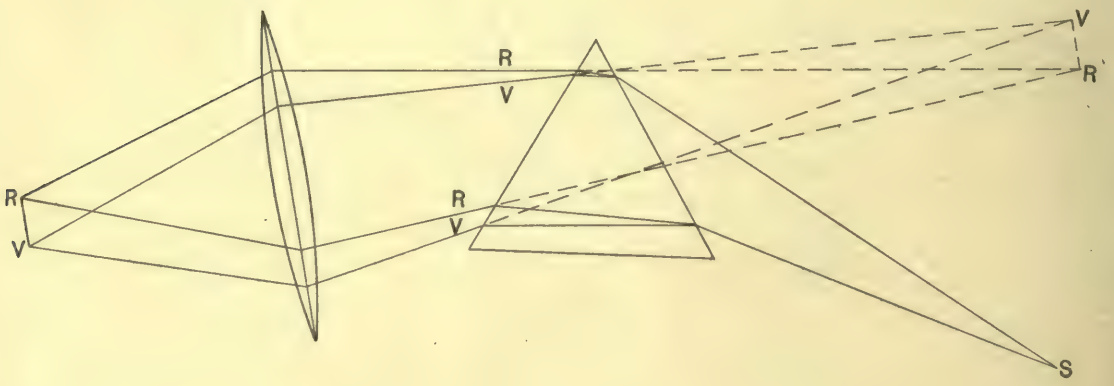
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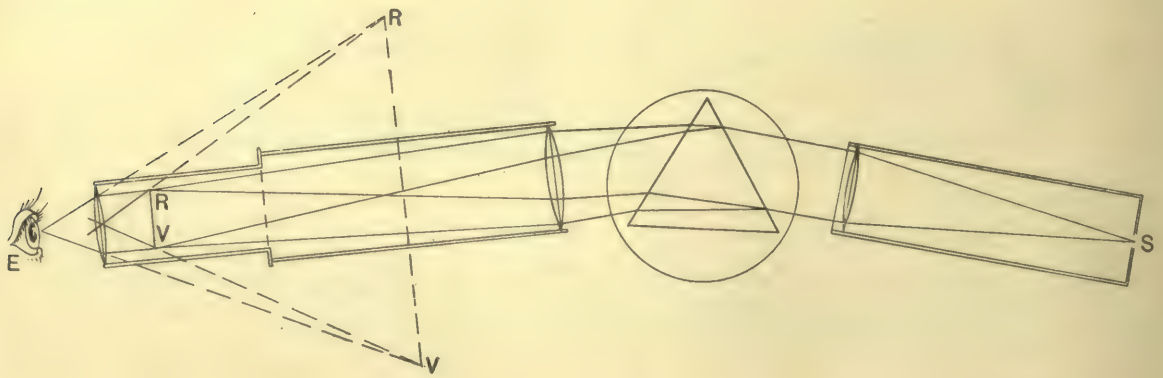
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Pure Spectrum

I. Pure Virtual Spectrum

If the screen is removed and the light transmitted by the prism is allowed to fall on the eye, each constituent of the sunlight appears to diverge from a separate virtual image of the illuminated aperture. Thus the violet image appears at one place and the red image at another. It is important to note the relative positions of red and violet.

If the mean path of the rays corresponds to that of *minimum deviation* through the prism, each coloured image will approximate to a point.

This is illustrated by the diagram on the previous page.

II. Pure Real Spectrum

1. Place a lens between the aperture and the prism. The red rays converge to one point and the violet to another.

2. Place a lens between the prism and the screen. The red rays converge to one point and the violet to another.

3. Spectrometer or Spectroscope.

(For a description of the instrument see "Prisms.")

The theory of the instrument is as follows:—

Rays diverging from the slit are rendered parallel by the collimator lens and fall as a parallel pencil on the prism.

Each colour will therefore form a parallel pencil having different deviation. These parallel pencils are brought to a focus in the focal plane of the telescope. The real image produced is viewed by the eye-piece, which gives a magnified virtual image.

Any particular part of the spectrum can be brought into coincidence with the intersection of the cross-wires.

To obtain a pure spectrum with the Spectroscope it is *not* necessary to have the prism in the position of minimum deviation.

SPECTRUM ANALYSIS

The first work was done by Brewster, Herschel and Talbot.

It had long been known that the salts of the alkali metals gave bright-line spectra owing to the dissociation of the chemical compounds by the heat of the flame. It was thought, however, that the same lines might be produced by different substances, especially as the sodium line was found in flames in which the metal was not supposed to be present. But Swan ascertained the presence of $\frac{1}{2500000}$ of a grain of sodium in a flame and he asserted that the line was always to be taken as an indication of the presence of sodium.

The Spectroscope thus gives a method of detecting the presence of various metallic substances. It has also led to the discovery of several new elements:—

e.g.	Caesium and Rubidium.	Bunsen.
	Thallium.	Crookes.
	Indium.	Reich and Richter.
	Helium.	Lockyer.

Different kinds of Spectra

I. Bright-line Spectra

The spectrum of an incandescent gas or vapour consists of a finite number of bright lines, separated by dark lines, *e.g.* the spectrum of sodium vapour.

Sodium gives two bright yellow lines termed the "D" lines, separated by a dark interval, depending upon the dispersive power of the prism and the magnifying power of the telescope. The two lines are really two images of the slit, which correspond to rays of two different wave-lengths.

(Incandescent = white, glowing or luminous with intense heat.)

II. Continuous Spectra

The spectrum of a glowing solid or liquid is continuous, *e.g.* the spectrum of carbon or platinum. It presents the appearance of an unbroken luminous band, varying in colour from point to point, and shading off on both sides of a certain point at which the intensity is a maximum.

The spectrum of a gas, candle or oil lamp flame is continuous. This is due to the fact that nearly all the light is emitted by incandescent particles of solid carbon. This is also the cause of the continuous spectra in the electric incandescent light and arc light.

III. Channelled or Fluted Spectra

The spectrum of a non-metal consists of a number of broad luminous bands, sharply defined at one edge and shading off gradually at the other edge, *e.g.* the spectrum of sulphur. When examined by a Spectroscope of great dispersive power, each fluting is found to consist of a considerable number of lines closely packed towards the definite edge of the fluting, and more and more widely spaced as the blurred edge of the fluting is approached.

They are obtained, as a general rule, with compounds or elements in the molecular state, whilst line spectra are given by elements in the atomic state. Some substances, however, give both fluted and line spectra.

(Flute = channel or groove.)

IV. Absorption Spectra

When light is incident upon the surface of a *transparent* medium, part is reflected and part transmitted. When, however, light is incident upon a substance like lamp-black, it is neither reflected nor transmitted, but *absorbed*, and ceases to exist as light. Lamp-black absorbs waves of all wave-lengths except the very largest. This is called "general absorption."

Certain substances absorb light corresponding to a particular part of the spectrum, and transmit the remaining part unchanged. This is called "selective absorption." It produces black bands in the spectrum of the transmitted light, which are called "absorption bands"; *e.g.* a piece of ruby glass transmits red rays and absorbs the light corresponding to the rest of the spectrum.

We arranged the Spectroscope to give the continuous spectrum of the flame of a fish-tailed burner, interposed certain solutions in the path of the rays and found what rays were transmitted.

1. Gamboge yellow—yellow and green.
2. Prussian blue—blue and green.
3. Mixture of the two—green only.

The Solar Spectrum is a continuous spectrum crossed by dark lines due to the absorption of rays of certain wave-lengths. This phenomenon is also exhibited by the moon and planets.

The Solar Spectrum

The Solar Spectrum is interrupted by dark lines, which were first observed by Wollaston in 1802, and were carefully studied at a later date by Fraunhofer and called "Fraunhofer's lines." They were explained first by Kirchhoff, who observed that the dark lines coincided with the bright lines in the spectra of the elements. The lines are due to selective absorption.

According to the "Theory of Exchanges," every substance, which emits certain kinds of rays to the exclusion of others, absorbs the same kinds which it emits.

When an incandescent vapour, emitting only waves of certain wave-lengths and therefore having a spectrum of bright lines, is interposed between the observer and a very bright source of light giving a continuous spectrum, the vapour allows no rays of its own particular kind to pass. The light which comes to the observer therefore consists of:—

1. Transmitted rays, in which these particular kinds are wanting.
2. Rays emitted by the vapour itself, which are of the same kind as those which it has refused to transmit.

Whether the rays of a particular kind shall on the whole predominate or not, will depend upon the relative brightness of the two sources.

(1) If the two sources are at all comparable in brightness, the rays will be in excess, inasmuch as they constitute the *whole* light of the one and only a *minute fraction* of the other.

(2) If the source giving the continuous spectrum is much more powerful, the rays appear black by contrast with the much brighter background, which belongs to the continuous spectrum.

This can be demonstrated as follows:—

Illuminate the slit of a Spectroscope with the arc light, when a continuous spectrum is obtained. Interpose between the arc light and the slit a spirit lamp, the wick of which has been previously soaked in salt solution and then dried, so that the white light has to traverse the flame of the spirit lamp before reaching the slit. The black lines, corresponding to the position of the "D" lines, should be seen.

In the cases of the Solar Spectrum it is supposed that the sun's light comes from

1. An inner stratum, which gives a continuous spectrum.

2. An outer layer, which contains vapours which absorb particular rays and thus produce the dark lines.

The inner stratum may be solid, liquid or gas, for, as the pressure of a gas is increased, its bright lines broaden out into bands, which at length give a continuous spectrum.

Fraunhofer observed 576 such lines in the Solar Spectrum and many more have since been noted. Most of these have been found to correspond to lines in the spectra of elements present on the earth. Consequently we have good grounds for believing that the chemical constitution of the sun is similar to that of the earth.

In the case of Helium the dark line, "D_3," was observed by Lockyer in the Solar Spectrum before the element was discovered by Ramsay in Cleveite. Hence the name Helium.

Fraunhofer named 354 lines on a map of the spectrum, *e.g.* the "D" line for sodium, which are referred to as reference marks for the accurate specification of different portions of the spectrum. They are always in the same places as regards colour, but do not retain exactly the same relative distances from one another. This is because, if prisms of different materials are employed, different parts of the spectrum are unequally expanded.

Invisible portions of the Spectrum

There are portions of the spectrum which do not affect the retina of our eye and are therefore invisible.

1. The brightness of the Solar Spectrum is greatest between "D" and "E," *i.e.* in the yellow, orange and light green, and falls off gradually on both sides.

2. The heating effect increases from the violet to the red and continues to increase beyond the visible spectrum at the red end. Thus the ultra-red or infra-red rays are thermal. They are waves of *long wave-length.* To show this effect, a rock-salt or carbon bisulphide prism is used, as they are found to be transparent to the waves of long wave-length, whereas glass is largely opaque to them. A thermopile is also necessary.

3. If the spectrum is thrown upon photographic paper, the action is usually insensible in the red, strong in the blue and violet, and sensible to a great distance beyond the violet end. Thus the ultra-violet rays are actinic. They are waves of *short wave-length.* To show this effect, a quartz prism is used, as it is found to be transparent to the waves of short wave-length, whereas glass is largely opaque to them. It has been found that atmospheric air absorbs the ultra-violet rays, and experiments on them have therefore to be conducted in vacuo. The ultra-violet Solar Spectrum is continuous and also comprises a number of dark lines.

This implies that the spectrum is composed of a large number of rays of different wave-length, of which only some affect the retina of our eyes.

Displacement of Lines consequent on Celestial Motions

According to the undulatory theory of light the fundamental difference between the different rays, which compose the complete spectrum, is a difference of wave-frequency and, in any given medium, the rays of *greatest* wave-frequency and *shortest* wave-length are the *most* refrangible.

Now in the case of sound it can be shown, by "Doppler's principle," that

if v = velocity,
n = frequency,
l = wave-length.

1. For approach.

n is greater and, since $v = nl$,

$$l = \frac{v}{n}.$$

Now, if n is greater, l will be smaller.

This is equally applicable in the case of light.

Since the wave-length is less, the refrangibility is greater, *i.e.* a line is displaced *towards* the violet end of the spectrum.

2. For recession.

Exactly the reverse is the case.

Thus displacement of a line towards the violet end of the spectrum indicates *approach*, whilst displacement in the other direction indicates *recession*. The velocity of approach and recession may also be calculated from the observed displacement.

When the slit of the Spectroscope covers a spot on the sun's disc, the dark lines lose their straightness in this part and are bent sometimes to one side and sometimes to the other. This indicates uprush and downrush of gases in the sun's atmosphere in the region occupied by the spot.

Huggins detected the displacement of the "F" line towards the red end in the spectrum of Sirius, as compared with the spectrum of the sun, indicating a motion of 30 miles per sec.

The stars Sirius, Rigel, Castor and Regulus are moving away from the Earth, whilst Arcturus and Vega are moving towards the Earth.

A similar displacement results from the rotation of the sun on its axis. One part of the sun's equator is approaching the observer and the opposite part is receding. The spectra of these two parts show opposite displacements of the dark lines.

COLOUR

Colours of Opaque Bodies

If a piece of red cloth or a red poppy is held in the red part of the spectrum, it appears red. If it is held in the green or blue part of the spectrum, it appears black. This shows that a body which is red in daylight is so because it reflects red rays only. Since it can reflect no green or blue rays, it appears dark in the green or blue light.

Speaking generally, the colour of an opaque body depends upon the mixture of rays which it reflects.

When white light, which is made up of many colours, falls upon a red surface, only that part of it which is red is reflected. The other colours are absorbed by the surface.

Colours of Transparent Bodies

A transparent body, seen by transmitted light, is coloured if it is more transparent to some rays than to others. No new ingredient is added, but certain ingredients are more or less completely absorbed. This is called "selective absorption."

A plate of red glass lets only red rays through and so appears red. A plate of blue glass lets only blue rays through and so appears blue. If two plates of glass, one red and the other blue, are placed one behind the other, the light that passes through has undergone a double process of selective absorption and therefore consists of those rays which are transmitted by *both* glasses. The colour which we see, in looking through the combination, is the part that is common to both. The combination of red and blue glass is practically black.

Colours of Powders

Each particle of a coloured powder may be regarded as a small transparent body, which colours light by selective absorption. Thus, when light falls on it, part is reflected and part transmitted. The light that is transmitted undergoes reflection at some of the surfaces of separation between the particles.

A single plate of uncoloured glass reflects $\frac{1}{25}$ of the light that falls on it, two plates $\frac{1}{13}$ and a large number nearly the whole. In a powder made of such a glass only $\frac{1}{25}$ is reflected from the first surface and that which gives the powder its colour comes by reflection from the lower layers.

The light is more coloured the more deeply it has penetrated. This is why coarse powder is darker than fine powder.

The quantity of light returned at each reflection depends only upon the number of reflections and *not* upon the thickness, but the intensity of the colour depends upon the depth to which the light has penetrated.

Mixtures of Colours

By the colour resulting from the mixture of two lights we mean the colour which is seen when they both fall on the same part of the retina. It is a *subjective* effect, *i.e.* one derived from our own consciousness.

The *objective* differences of colour are differences of wave-frequency or period of vibration. All the colours in a pure spectrum are accordingly *objectively* simple, each having its own definite period of vibration, which distinguishes it from all others.

Different compositions *objectively* may produce precisely the same visual impression *subjectively*. In this way light differs from sound. Every colour that we see in nature can be exactly imitated by an infinite variety of different combinations of elementary light. To the naked eye all these whites are completely indistinguishable from one another.

Primary Colours

Def.

"A primary colour is one which cannot be imitated to the eye by the mixture of any other colours."

Maxwell showed that there are *three* primary colours, red, green and *violet*, and that any other colour can be formed by mixing suitable proportions of these.

Methods of Mixing Colours

1. By combining reflected and transmitted light, *e.g.* at a window. Hold inside the window the brighter of two objects, and outside the window the second object in such a position that it coincides with the image of the first.
2. By employing a rotating disc composed of differently-coloured sectors.
3. By causing two or more spectra to overlap.

Complementary Colours

Any two colours which produce the sensation of white, when they are mixed, are said to be complementary.

e.g. yellow and blue.
green and purple.

Theories of Colour Vision

The commonly accepted theory of colour vision is that due to Young and Helmholtz. It states that, just as there are three primary colours, so there are *three primary colour sensations*, due to the existence of three distinct sets of nerves, having their termination in different parts of the retina. The three colours to which these nerves respond are red, green and *blue*. It will be noted that these are *not* the primary colours. According to this theory the colour of a body depends upon the proportion in which the three sets of nerves are excited.

After-Images

If we look steadily at a bright stained-glass window and then turn our eyes to a white wall, we see an image of the window with the colours changed into their complementaries. The explanation is that the nerves, which have been strongly exercised in the perception of the bright colours, have had their sensibility diminished, so that the balance of action necessary for the sensation of white no longer exists and those elements, which have not been weakened, preponderate. The *subjective* appearances arising have been called "negative after-images." There are sometimes "positive after-images" followed by "negative after-images." The former is a case of persistence of impressions.

Colour-Blindness

If one set of nerves is absent or inactive, the person is said to be colour-blind. The commonest type of colour-blindness is *red-blind, i.e.* the red constituent of all colours is unappreciated. The vision is *dichroic* instead of *trichroic.*

Recomposition of White Light

Just as it is possible to decompose white light into its constituents, so it is possible to recombine them into white light.

1. By the use of a second prism exactly like the first, but with its refracting edge turned in the opposite direction.

2. By interposing an achromatic lens in the course of the rays which form the spectrum.

3. By Newton's disc, which is a circular piece of cardboard divided into sectors, painted with the colours of the spectrum, and attached to a whirling table. When the disc is rapidly rotated, it appears nearly white, not because there is any real mixture of colours, but owing to the persistence of impressions. This is a *physiological*, though not a *physical*, blending of the colours.

4. By combining complementary colours.

Rainbows

Magnifying Power

Chromatic Aberration

Spherical Aberration

Wave Theory of Light

Interference

Diffraction

Polarisation

(See advanced notes.)

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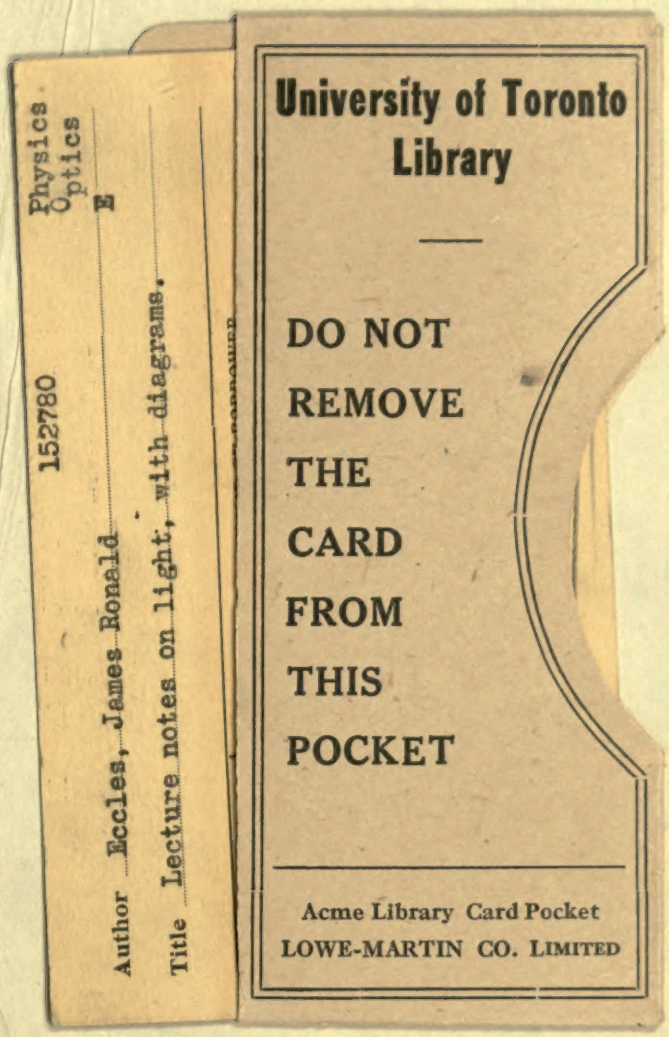
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